On the Robustness of Algebraic STBCs to Coefficient Quantization

J. Harshan
Dept. of Electrical and Computer Systems Engg.,
Monash University
Clayton, VIC 3800, Australia
Email:harshan.jagadeesh@monash.edu

Emanuele Viterbo
Dept. of Electrical and Computer Systems Engg.,
Monash University
Clayton, VIC 3800, Australia
Email:emanuele.viterbo@monash.edu

***Abstract*— In this paper, we study the robustness of high-rate algebraic space-time block codes (STBCs) to coefficient quantization (CQ) at the transmitter in 2 $\times$ 2 MIMO fading channels. In particular, we investigate the impact of CQ on the bit error rates of the Golden code and the Silver code with M-QAM constellations. We assume infinite-precision operations at the receiver. Towards generating the matrix codewords, we find the minimum number of bits needed to *represent* and perform the *arithmetic operations* such that the finite-precision versions of these codes provide BER close to their infinite-precision counterparts. We show that the Golden code and the Silver code suffer very slight degradation in the BER performance so long as at least 7-bits and 6-bits are used for 4-QAM constellations, respectively. Also, both codes are shown to need a minimum of 8-bits for 16-QAM constellation. Finally, we propose an example of a full-rate, full-diversity STBC which can be encoded with as low as 3-bits for 4-QAM constellation. The advantages of the proposed code are also discussed.**

I. Introduction and Preliminaries

Golden code [1]-[3] and Silver code [4]-[6] are two well known full-rate, full-diversity Space-Time Block codes (STBCs) for 2 $\times$ 2 Multiple-Input Multiple-Output (MIMO) fading channels. These codes provide a rate of 2 complex-symbols per channel use by utilizing all the degrees of freedom in the MIMO channel. Importantly, these codes can be decoded using the *universal lattice-decoder* [7], [8] and hence, can provide optimal error performance with reasonable complexity at the receiver.

In practice, it is preferred to implement wireless transceivers with limited hardware and/or processors operating on fewer bits. In order to implement high-rate STBCs on transceivers with limited hardware, it is interesting to study the robustness of high-rate STBCs to finite-precision operations at both the transmitter and the receiver. For the performance of some 2 $\times$ 2 high-rate algebraic STBCs to finite-precision operations at the receiver, we refer the reader to [9], [10] and the references within. We are not aware of any work that study the error performance of high-rate STBCs to finite-precision *encoding operations* at the transmitter. A study along those lines is particularly important for high-rate STBCs since, the underlying encoding operation involves computation of linear combinations of real information symbols with *irrational* coefficients. The presence of irrational coefficients is due to the construction method which is based on algebraic extensions [1], [5]. Therefore, when STBCs from high-rate algebraic designs are implemented with limited hardware, there could be substantial degradation in the error-performance due to the quantization of the irrational coefficients (referred to as coefficient quantization [11]) and finite-precision encoding operations.

In this work, we study the robustness of the 2 $\times$ 2 STBCs such as the Golden code and the Silver code to the coefficient quantization and finite-precision operations at the transmitter. The contributions and the organization of this work are given below:

- We find the robustness of the bit error rate (BER) of the Golden code and the Silver Code to coefficient quantization and fixed-point encoding operations at the transmitter. The receiver is assumed to work with infinite-precision and the decoder is assumed to have the knowledge of the quantized codebook (Section II).
- We determine the *minimum* number of bits needed to encode the Golden code and the Silver code such that fixed-point versions of these codes provide the BER close to their infinite-precision counterparts. For 4-QAM constellation, we show that the Silver code needs 6 bits and the Golden code needs 7 bits to approximately retain their BER performance. For 16-QAM constellation, both the codes are shown to need 8 bits (Section III).
- The minimum number of bits needed for these codes is confirmed by displaying a two-dimensional grid of the set of all complex-symbols transmitted from the finite-precision encoder, and comparing them with the output of the infinite-precision encoder. Through computer simulations, BER of the two codes are also obtained for various number of bits to justify the results (Section III).
- Finally, we propose a new full-rate, full-diversity STBC which needs only 3-bit encoding operations at the transmitter. With only 3-bits, the proposed code outperforms the Golden code and Silver code since the latter codes experience error-floors. We also compare the BER of the proposed code with infinite-precision versions of the latter codes, and discuss the advantages of the proposed code (Section IV).

***Notations**:* Throughout the paper, boldface letters and capi-

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tal boldface letters are used to represent vectors and matrices, respectively. For a complex matrix $\mathbf{X}$, the matrices $\mathbf{X}^*$, $\mathbf{X}^T$, $\mathbf{X}^H$, $|\mathbf{X}|$, $\mathbf{X}_I$ and $\mathbf{X}_Q$ denote, respectively, the conjugate, transpose, conjugate transpose, determinant, real part and imaginary part of $\mathbf{X}$. For a matrix $\mathbf{X}$, we use $\mathbf{X}(i,j)$ to represent the element in the i-th row and the j-th column of $\mathbf{X}$. Similarly, for a vector $\mathbf{x}$, we use $\mathbf{x}(i)$ to denote the i-th element of $\mathbf{x}$. The set of all integers, the real numbers, and the complex numbers are, respectively, denoted by $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$ and $\imath = \sqrt{-1}$. Cardinality of a set $\mathcal{S}$ is denoted by $|\mathcal{S}|$. Absolute value of a complex number x is denoted by $|x|$ and $E[\cdot]$ denotes the expectation operator. A circularly symmetric complex Gaussian random vector, $\mathbf{x}$ with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Gamma}$ is denoted by $\mathbf{x} \sim \mathcal{CG}(\boldsymbol{\mu}, \boldsymbol{\Gamma})$.

II. Signal Model

The 2×2 MIMO channel considered consists of a source and a destination terminal each equipped with 2 antennas. For $i, j \in \{1, 2\}$, the channel between the i-th transmit antenna and the j-th receive antenna is assumed to be flat fading and hence, denoted by the complex number $h_{i,j}$. Each $h_{i,j}$ remains constant for a block of T ($T \geq 2$) complex channel uses and is assumed to take an independent realization in the next block. Statistically, we assume $h_{i,j} \sim \mathcal{CG}(0,1)\ \forall i, j$. The source conveys information to the destination through a 2×2 STBC denoted by $\mathcal{C}$. We assume that a linear design $\mathbf{X}_{\mathcal{LD}}$ [12] in complex variables $x_1, x_2, \cdots x_k$ is used to generate $\mathcal{C}$ by making them take values from an underlying complex constellation $\mathcal{S}$. Throughout this work, we use both the Golden code design and the Silver code design as $\mathbf{X}_{\mathcal{LD}}$, and regular M-QAM constellations as $\mathcal{S}$. If $\mathbf{X} \in \mathcal{C}$ denotes a transmitted codeword matrix such that $E[|\mathbf{X}(i,j)|^2] = 1\ \forall i, j$, then the received matrix $\mathbf{Y} \in \mathbb{C}^{2\times 2}$ at the destination is given by,

$$\mathbf{Y} = \sqrt{\frac{\rho}{2}}\mathbf{H}\mathbf{X} + \mathbf{N}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{2\times 2}$ denotes the channel matrix, $\mathbf{N} \in \mathbb{C}^{2\times 2}$ denotes the AWGN at the destination such that $\mathbf{N}(i,j) \sim \mathcal{CG}(0,1)\ \forall i, j$ and ρ denotes the average receive Signal-to-Noise Ratio (SNR) per receive antenna. We assume a coherent MIMO channel where only the receiver has the complete knowledge of $\mathbf{H}$ (the transmitter does not know $\mathbf{H}$). In the following subsection, we recall the structure of the Golden code and the Silver code designs.

A. Representation of Golden and Silver code design

To facilitate the study of finite-precision operations at the transmitter, we consider the following representation of the Golden code design denoted by $\mathbf{X}_G$ as,

$$\mathbf{X}_G = \begin{bmatrix} g_{1,I} + \imath g_{1,Q} & g_{3,I} + \imath g_{3,Q} \\ g_{2,I} + \imath g_{2,Q} & g_{4,I} + \imath g_{4,Q} \end{bmatrix}, \tag{2}$$

where

$$\mathbf{g} = [g_{1,I}\ g_{1,Q}\ g_{2,I}\ g_{2,Q}\ \cdots\ g_{4,I}\ g_{4,Q}]^T \in \mathbb{R}^{8\times 1}$$

is given by $\mathbf{g} = \mathbf{R}_G\mathbf{x}$ where $\mathbf{R}_G$ is given in (3) at the top of the next page, and

$$\mathbf{x} = [x_{1,I}\ x_{1,Q}\ x_{2,I}\ x_{2,Q}\ x_{3,I}\ x_{3,Q}\ x_{4,I}\ x_{4,Q}]^T \in \mathbb{R}^{8\times 1}.$$

Also, $\theta = \frac{(1+\sqrt{5})}{2}$ and $\sigma(\theta) = \frac{(1-\sqrt{5})}{2}$ are the Golden code parameters. Note that the components of $\mathbf{x}$ take values from the $\sqrt{M}$-PAM constellation. Along the similar lines, Silver code design is represented by

$$\mathbf{X}_S = \begin{bmatrix} s_{1,I} + \imath s_{1,Q} & s_{3,I} + \imath s_{3,Q} \\ s_{2,I} + \imath s_{2,Q} & s_{4,I} + \imath s_{4,Q} \end{bmatrix}, \tag{4}$$

where $\mathbf{s} = \mathbf{R}_S\mathbf{x}$ and $\mathbf{R}_S$ is given in (5) at the top of the next page such that $u_1 = \frac{1}{\sqrt{7}}(1+\imath), u_2 = \frac{1}{\sqrt{7}}(-1+2\imath), u_3 = \frac{1}{\sqrt{7}}(1+2\imath)$ and $u_4 = \frac{1}{\sqrt{7}}(1-\imath)$. In the next subsection, we discuss the encoding operation for the Golden code and the Silver code emphasising the need for matrix multiplication operation in the encoder.

B. Encoding operations for Golden code and Silver code

To encode a space-time codeword, we assume that binary digits arrive in a block of $L = 8[\log_2(\sqrt{M})]$ bits. These L bits are partitioned into 8 sub-blocks of $\log_2(\sqrt{M})$ bits, and each sub-block is gray mapped to a point in the $\sqrt{M}$-PAM constellation. After gray mapping, the resulting set of 8 real values corresponds to the vector $\mathbf{x}$. Further, the vector $\mathbf{x}$ is multiplied by the fixed matrix $\mathbf{R}_G$ (or $\mathbf{R}_S$) to obtain $\mathbf{g} = \mathbf{R}_G\mathbf{x}$ (or $\mathbf{s} = \mathbf{R}_S\mathbf{x}$). Finally, the components of $\mathbf{g}$ (or $\mathbf{s}$) are represented in the matrix form as in (2) (or (4)) to obtain a matrix codeword.

To transmit every block of L bits, the encoder has to perform *real* matrix multiplication of $\mathbf{x}$ and $\mathbf{R}_G$ (or $\mathbf{R}_S$). For the Golden code, every row of $\mathbf{R}_G$ has 4 non-zero entries and hence, to obtain each component of $\mathbf{g}$, the Golden code encoder needs a total of 32 scalar multiplications and 24 real additions. For the Silver code, every row of $\mathbf{R}_S$ has 5 non-zero entries and hence, the Silver code encoder needs 40 scalar multiplications and 32 real additions. It is important to note that some entries of $\mathbf{R}_G$ and $\mathbf{R}_S$ are *irrational* numbers. Therefore, in order to retain the performance of these STBCs, the matrix multiplication, when performed with finite-precision should retain the precision in the components of $\mathbf{g}$ and $\mathbf{s}$.

In the following section, we discuss the finite-precision encoding operations of the Golden code and the Silver code design. We first discuss the finite-precision representation of $\mathbf{R}_G$, $\mathbf{R}_S$ and $\mathbf{x}$, and subsequently, discuss the finite-precision matrix multiplication operation.

III. Performance of Golden code and Silver Code with Finite-Precision Encoders

In this section, we first discuss the finite-precision representation of $\mathbf{R}_G$, $\mathbf{R}_S$ and $\mathbf{x}$ using $(q-1)$-bits for the fractional part and 1-bit for the sign part. For any $y \in \mathbb{R}$ such that $|y| < 1$, we define the q-bit quantized version of y as

$$Q_q(y) \triangleq \frac{[y2^{q-1}]}{2^{q-1}}, \tag{6}$$

$$\mathbf{R}_G = \begin{bmatrix} 1 & -\sigma(\theta) & \theta & 1 & 0 & 0 & 0 & 0 \\ \sigma(\theta) & 1 & -1 & \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\theta & -1 & 1 & -\sigma(\theta) \\ 0 & 0 & 0 & 0 & 1 & -\theta & \sigma(\theta) & 1 \\ 0 & 0 & 0 & 0 & 1 & -\sigma(\theta) & \theta & 1 \\ 0 & 0 & 0 & 0 & \sigma(\theta) & 1 & -1 & \theta \\ 1 & -\theta & \sigma(\theta) & 1 & 0 & 0 & 0 & 0 \\ \theta & 1 & -1 & \sigma(\theta) & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\mathbf{R}_S = \begin{bmatrix} 1 & 0 & 0 & 0 & u_{1,I} & -u_{1,Q} & u_{2,I} & -u_{2,Q} \\ 0 & 1 & 0 & 0 & u_{1,Q} & u_{1,I} & u_{2,Q} & u_{2,I} \\ 0 & 0 & 1 & 0 & -u_{3,I} & u_{3,Q} & -u_{4,I} & u_{4,Q} \\ 0 & 0 & 0 & 1 & -u_{3,Q} & -u_{3,I} & -u_{4,Q} & -u_{4,I} \\ 0 & 0 & -1 & 0 & -u_{3,I} & u_{3,Q} & -u_{4,I} & u_{4,Q} \\ 0 & 0 & 0 & 1 & u_{3,Q} & u_{3,I} & u_{4,Q} & u_{4,I} \\ 1 & 0 & 0 & 0 & -u_{1,I} & u_{1,Q} & -u_{2,I} & u_{2,Q} \\ 0 & -1 & 0 & 0 & u_{1,Q} & u_{1,I} & u_{2,Q} & u_{2,I} \end{bmatrix} \tag{5}$$

where $[x]$ represents the nearest integer to x. Henceforth, throughout the paper, we use fixed-point operations for finite-precision operations. To avoid quantization errors due to *over-flows*, we scale down the values of $\mathbf{R}_G$ and $\mathbf{x}$ using suitable scale factors such that each component of $\mathbf{R}_G$, $\mathbf{x}$ and $\mathbf{R}_G\mathbf{x}$ fall within the quantization interval $[-1, 1)$.

With the above fixed-point representation, the q-bit encoding operation is only subject to quantization errors due to rounding operations in the least significant bits. Once the components of $\mathbf{R}_G$, $\mathbf{R}_S$ and $\mathbf{x}$ have q-bit representations, the matrix multiplication operation involves q-bit scalar multiplications and a series of q-bit scalar additions. In particular, the output of the scalar multiplier is restricted to q-bits using the rounding operation. Therefore, for the fixed-point encoder, the source of errors are

- the errors in quantizing each component of $\mathbf{R}_G$ (or $\mathbf{R}_S$) and
- the errors in quantizing the output the scalar-multiplier.

Note that there are no errors from the scalar-adder as $\mathbf{R}_G$, $\mathbf{R}_S$ and $\mathbf{x}$ are appropriately scaled to avoid over-flows. In particular, the matrices $\mathbf{R}_G$ and $\mathbf{R}_S$ given in (3) and (5) respectively, are scaled down by a factor of 2. Also, for both these codes, we scale the entries of $\mathbf{x}$ such that $\mathbf{x}(i) \in \frac{1}{2}\{-1, 1\}$ $\forall i$ for 4-QAM constellation, and $\mathbf{x}(i) \in \frac{1}{8}\{-3, -1, 1, 3\}$ $\forall i$ for 16-QAM constellation. Henceforth, the encoder with infinite-precision is referred to as Infinite-Precision Encoder (IPE). Similarly, encoder with fixed-point operations is referred as Fixed-Point Encoder (FPE).

In Fig.1-Fig.4, we present the set of all possible complex transmitted signal points obtained at the output of the FPE using 4-QAM constellation. In other words, these are all possible entries $\mathbf{X}(i, j)$ of the space-time codewords of a given code. For more clarity, the output of both encoders are scaled by 2^{q-1} and then presented in Fig.1-Fig.4. For the Golden code, the transmitted signal points are presented in Fig. 1 and Fig. 2 for $q = 6$ and $q = 7$, respectively. Similar points are also presented using Silver code in Fig. 3 and Fig. 4 for $q = 5$ and $q = 6$, respectively. Note that

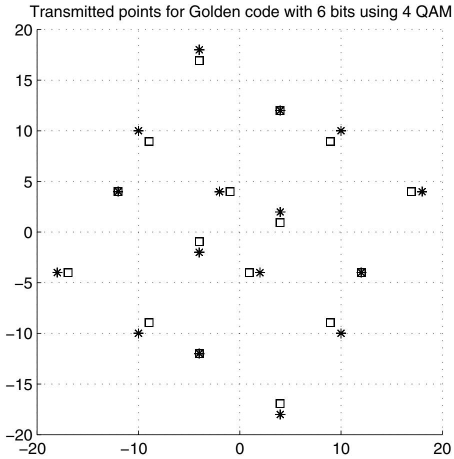

Fig. 1. Transmitted points for Golden code with 6 bits. Symbol $*$ and symbol $\square$ denotes the output of FPE and IPE, respectively.

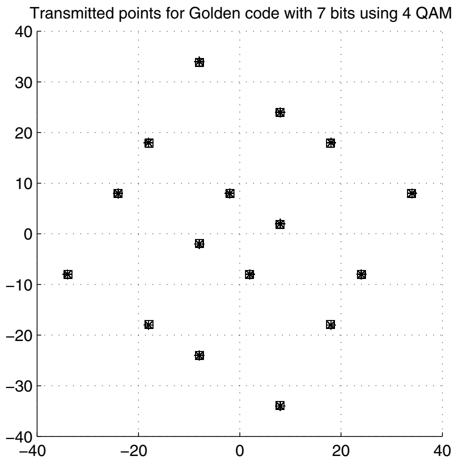

Fig. 2. Transmitted points for Golden code with 7 bits. Symbol $*$ and symbol $\square$ denotes the output of FPE and IPE, respectively.

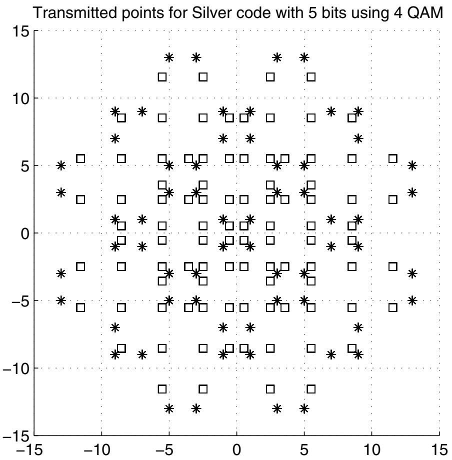

Fig. 3. Transmitted points for Silver code with 5 bits. Symbol $*$ and symbol $\square$ denotes the output of FPE and IPE, respectively.

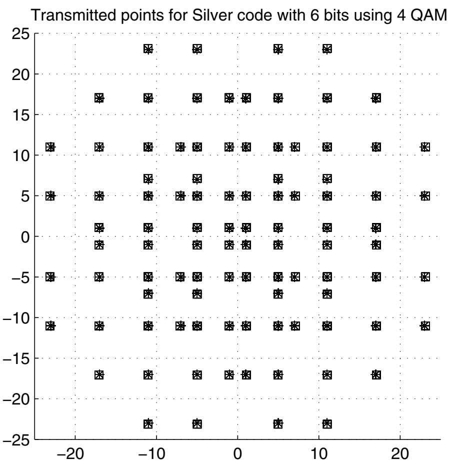

Fig. 4. Transmitted points for Silver code with 6 bits. Symbol $*$ and symbol $\square$ denotes the output of FPE and IPE, respectively.

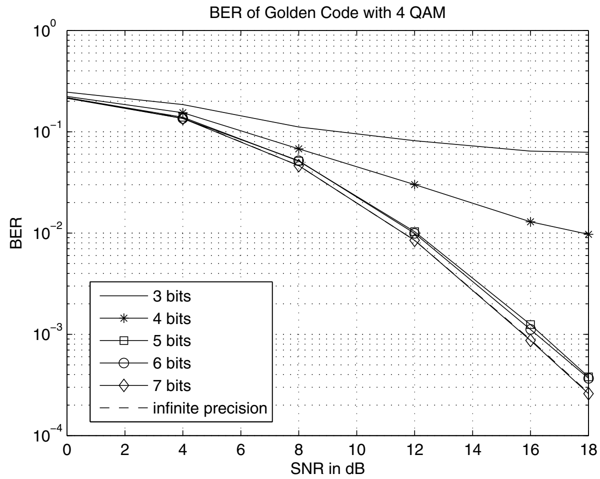

Fig. 5. BER of Golden code with 4-QAM constellation for various q-bit encoders

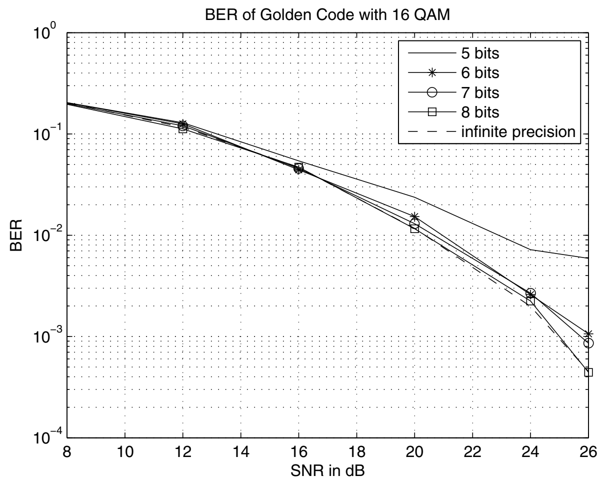

Fig. 6. BER of Golden code with 16-QAM constellation for various q-bit encoders

we present the transmitted points for two different values of q to highlight the *matching* and *mismatching* of the transmitted points between the outputs of the FPE and IPE. We observe that for $q = 7$ and $q = 6$, the transmitted points of the FPE and IPE approximately coincide for Golden code and Silver code, respectively. Similar plots can be obtained for smaller values of q to observe the mismatch between the two encoders. For 16-QAM constellation, it can be verified that the set of transmitted points start to coincide with $q = 8$ for both the codes.

In Fig.5-Fig.8, we present the BER performance of the Golden code and the Silver code as a function of the receive SNR ρ. In particular, for each code, we compare the error-performance of their quantized codebooks for various values of q with 4-QAM and 16-QAM constellations. At the receiver, the corresponding q-bit quantized version of $\mathbf{R}_G$ and $\mathbf{R}_S$ are used in the sphere-decoding algorithm. From the figures, the BER results of the codes with FPE and IPE are shown to approximately match, when sufficient number of bits are used at the encoder. The minimum number of bits required to match the BER are observed to be the same as pointed out using the rectangular grid of transmitted points. Note from Fig. 5 and Fig. 7 that, for very low number of bits, the two codes experience error-floors since the transmitted points are not uniquely distinguishable.

IV. A full-rate, full-diversity STBC with 3-bit encoder

In the previous section, we have shown that the Golden code and the Silver code respectively need at least 7-bit and 6-bit encoding operations for 4-QAM constellation. Such a high value of the minimum number of bits can be attributed to the presence of irrational coefficients in the code structure. In this section, we present an example of a full-rate, full-diversity STBC which needs only 3-bit encoding operations.

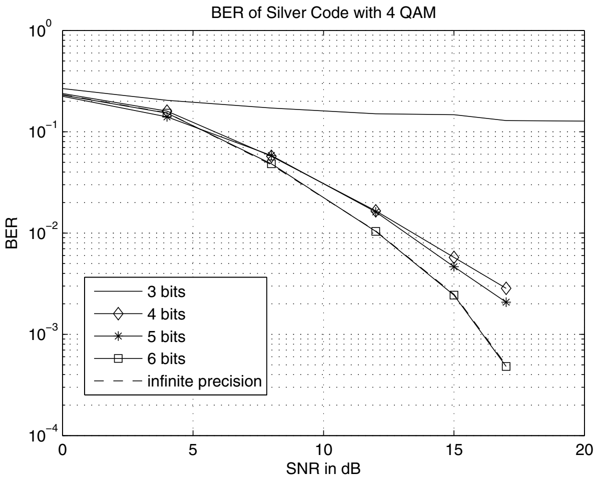

Fig. 7. BER of Silver code with 4-QAM constellation for various q-bit encoders

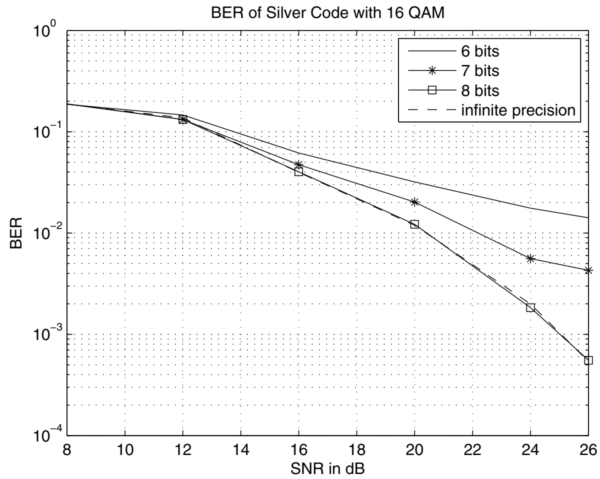

Fig. 8. BER of Silver code with 16-QAM constellation for various q-bit encoders

This code was found by an exhaustive search among all 2×2 codes with integer coefficients, and has the largest minimum determinant in the search space. The proposed code does not have irrational numbers as coefficients, thus can be exactly represented in finite-precision. We refer to this code as the integer code (IC), defined as

$$\mathbf{X}_I = \begin{bmatrix} (x_1 + \alpha x_2) & (x_3 + \bar{\alpha} x_4) \\ \gamma(x_3 + \alpha x_4) & (x_1 + \bar{\alpha} x_2) \end{bmatrix} \bmod 8, \qquad (7)$$

where $\gamma = \imath$, $\alpha = 2\imath$ and $\bar{\alpha} = 6\imath$. The variables x_i take values from the constellation $\mathcal{S} = \{0, 3, 3\imath, 3 + 3\imath\}$. Because of the modulo operation, we have $\mathbf{X}_I(i, j) \in \mathbb{Z}_8[\imath]\ \forall i, j$ where $\mathbb{Z}_8 = \{0, 1, 2, \cdots, 6, 7\}$ is the finite ring of integers modulo 8. Though, the entries of the codeword belong to $\mathbb{Z}_8[\imath]$, an appropriately scaled and translated version of $\mathbf{X}_I$ given by,

$$\mathbf{X}_I^{'} = 2\mathbf{X}_I - (7 + \imath 7) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad (8)$$

is transmitted. Hence, the transmitted points of the IC take values from a regular 64-QAM constellation.

Since all the coefficients of the IC and the entries of $\mathcal{S}$ belong to $\mathbb{Z}_8[\imath]$, the fixed-point encoding operations can be performed with only 3 bits for each in-phase and the quadrature component. Along the similar lines of the Golden code and Silver code, linear combinations of the information symbols are transmitted through each transmit antenna, however through a modulo operation. Note that the modulo operation is equivalent to dropping higher order bits in fixed-point operations, and hence, the IC can be conveniently implemented with 3 bits.

In Fig. 9, we present the BER performance of the IC along with the 3-bit versions and infinite-precision versions of the Golden code and the Silver code (when used with 4-QAM constellation). Note that all the three codes transmit at 4 information bits per channel use. From the figure, the IC is shown to provide full-diversity as its BER curve moves parallel

TABLE I

MINIMUM SQUARED DETERMINANT VALUES FOR VARIOUS q-BIT ENCODERS

q | code	Golden code	Silver code	Integer code
3	0	0	0.0022
4	0	0.0083	-
5	0.0494	0.0263	-
6	0.0494	0.1429	-
7	0.1878	0.1429	-
⋮	⋮	⋮	-
∞	$\frac{1}{5}$	$\frac{1}{7}$	-

to those of the known full-diversity codes. The full-diversity property of the IC can also be verified by checking the minimum-rank criterion. In Table I, we present the minimum squared determinant (MSD) values of the difference matrix of the codeword matrices of the quantized codebooks for different q-values. The numbers presented in Table I show that the MSD values for $q = 7$ and $q = 6$ for the Golden code and the Silver Code respectively, are very close to their infinite-precision values. We point out that the BER curves in Fig.5-Fig.8 are insensitive to this marginal difference in the MSD values, since the MSD is a relevant metric only at very large SNR values. If we use 3-bit encoders for all three codes, then the integer code will outperform the the other two codes since the latter ones do not provide full diversity with 3 bits. However, when compared with infinite-precision versions, the IC is shown to be poorer by about 3 dB round a BER of 10^{-3}. Note that the IC comes at the cost of some SNR loss but with the advantage of 4-bits less than the Golden code for the encoder. However, IC is not a linear dispersion code [12], since $\mathbf{X}_I$ cannot be expressed as $\sum_{i=1}^{4}(x_{i,I}A_{i,I} + x_{i,Q}A_{i,Q})$ for some $A_{i,I}, A_{i,Q} \in \mathbb{C}^{2\times 2}$. This disadvantage makes the proposed code non-sphere decodable and hence, we have used brute-force ML decoder to obtain the BER results. By presenting this example, we have shown the existence of a full-rate, full-diversity code with only 3-bit

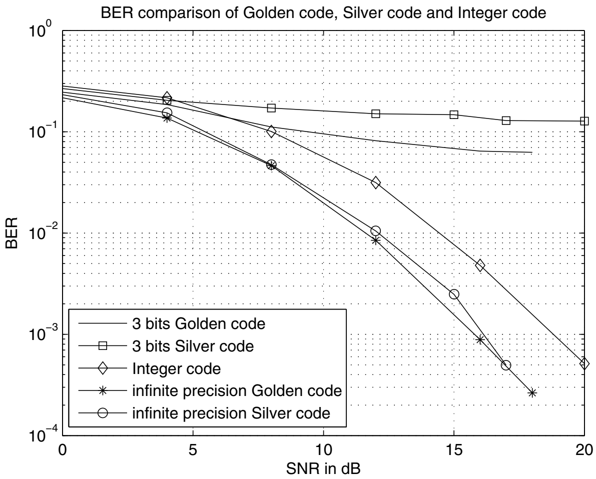

Fig. 9. BER comparison among the Golden code, Silver code and Integer code with 4-bits per channel use.

encoding operations.

V. Summary and Directions for Future Work

In this work, we have studied the robustness of some 2 $\times$ 2 algebraic STBCs to coefficient quantization and fixed-point encoding operations at the transmitter. We have shown that the Golden code and the Silver code need at least 7 and 6 bit-encoding operations, respectively to retain the error performance of their infinite-precision versions. Simulation and numerical results are presented to justify the required minimum number of bits. We also propose an example of a full-rate, full-diversity STBC with only 3-bits for encoding operations. An interesting direction for future work is to obtain a systematic construction of high-rate STBCs with reduced number of bits for the encoding operations. Another direction for future work is to design high-rate codes with integer coefficients and study their robustness to quantization effects at the receiver.

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