Optimal Placement of Wireless Nodes for Maximizing Path Lifetime

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***Abstract*— In this work we investigate the deployment of wireless nodes in order to maximize the lifetime of a data flow. We develop a mathematical model for determining the best placement of nodes by taking into consideration the energy of each node involved in the data flow. By using our mathematical model we achieve two major objectives: the maximization of the shortest node's lifetime and the convergence of all the nodes' lifetime to a unique value.**

***Index Terms*— Wireless sensor networks, lifetime.**

I. INTRODUCTION

THE placement of nodes in a wireless network is an important and growing research field since the energy consumption and the lifetime of a network rest on the power used in the transmission and reception. This power usage, in turn, depends on the mutual position of the pair of communicating nodes.

Toumpis and Tassiulas found a scalar nonlinear partial differential equation for determining the optimal positions of nodes in a massively dense sensor network, so that a minimum number of nodes is needed for a data flow [1]. The objective of their work is to find a tradeoff between the length of routes and the number of nodes in each route, without taking into consideration the energy consumption. In [2] the authors explore the problem of the optimal placement of Wireless Sensor Networks (WSN) devices, but they do not investigate the scenarios where the transmission range of nodes is adaptive nor do they take into account the optimal positions of the nodes in relation to their residual energies. Moreover, we know from [3] that a straight path, between source and destination, is most energy efficient and there is also a unique hop count for any distance that minimizes the cost of communications. Goldenberg *et al.* show in [4] that the optimal positions of the relay nodes must lie entirely on the line between the source and the destination, and these nodes must be evenly spaced along the line. Therefore, from now on, we shall refer to this approach as "evenly spaced".

In this letter we propose a mathematical model which focuses on the maximization of the lifetime of the path of nodes involved in a data flow. This model allows us to find the best placement of the devices when they have different levels of residual energies. Previous works cited did not place an emphasis on the importance of different levels of residual energies among their working assumptions, for this reason we called our approach "energy spaced". From the model we find out that the optimal placement is on the straight line between source and destination as in [4], but the nodes must be spaced according to their residual energies. When we compare our approach with the random and the evenly spaced deployments, the results show that the energy spaced solution achieves a much longer lifetime. In WSN, wasteful disconnections and inconvenient replacements of nodes are often caused by wide variations in the lifetime values of nodes. Our placement strategy avoids these problems by making all the nodes involved in the route last the same amount of time. To the best of our knowledge, no mathematical scheme based on the residual energy of nodes has been introduced for the placement of wireless devices to date.

II. SYSTEM AND MODEL DESCRIPTION

We consider a data flow between a source and a destination in a sensor field. The positions of the relay nodes have been chosen according to three different strategies of placement:

- random;
- evenly spaced along the straight line. This is according to [4] in order to minimize the energy consumption;
- energy spaced along the straight line, taking into account the different levels of residual energy of the relay nodes.

The last strategy is the focus of this letter, and it has been optimized as a result of the mathematical model which follows.

The energy model we used to characterize the physical layer of our mathematical scheme is taken from [5]. By simplifying this model we obtain that the energy required to send one bit at the distance d is $E = \beta d^{\alpha}$, where α is the exponent of the path loss ($2 \leq \alpha \leq 5$), β is a constant $[\mathrm{J}/(\mathrm{bit}\cdot\mathrm{m}^{\alpha})]$. We set α equal to 2 and β equal to 10 $\mathrm{pJ}/(\mathrm{bit}\cdot\mathrm{m}^2)$, which are typical values of a free space model.

Next, we introduce the mathematical model of the system. Let $\mathbf{v}_1$ and $\mathbf{v}_n$ denote the known source and destination positions, respectively. Let $\{\mathbf{v}_i\}_{i=2}^{n-1}$ be the positions of the $n-2$ relay nodes. Let $\{T_i\}_{i=1}^{n-1}$ and $\{E_i\}_{i=1}^{n-1}$ be the life times and the residual energies of the nodes, respectively. Let P_{rec} denote the minimum required power in order for a bit to be correctly received. We assume a power control system is in place so that the transmitter adjusts its power in order to deliver P_{rec} at the receiver. This implies that each T_i is a function of the positions of nodes i and $i+1$, i.e. $T_i = \frac{E_i}{P_{rec}\|\mathbf{v}_i - \mathbf{v}_{i+1}\|^2}$. The distance between two successive nodes in the path is $\|\mathbf{v}_i - \mathbf{v}_{i+1}\|$.

Problem: Find $\{\mathbf{v}_i\}_{i=2}^{n-1}$ such that $\min\{T_i\}_{i=1}^{n-1}$ is maximized. This can be immediately solved by placing the nodes on the segment with the extremes $\mathbf{v}_1$ and $\mathbf{v}_n$, the distance between adjacent nodes being chosen in order to have $T_1 = T_2 = \cdots =$

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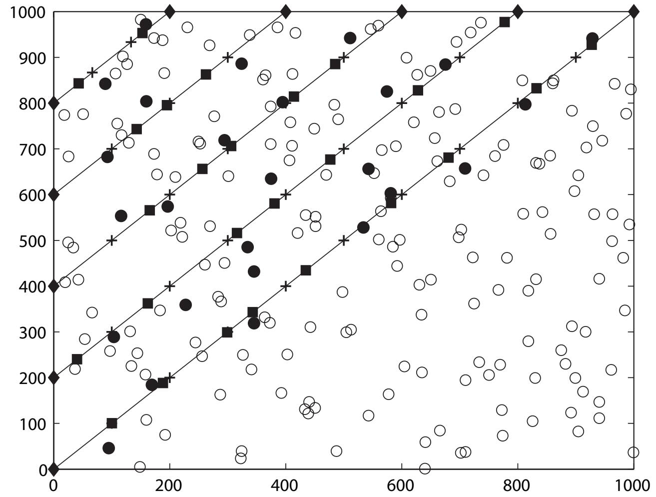

Fig. 1. Different data flows considered.

TABLE I

EVALUATION PARAMETERS

Field Area (LxL)	1000 m x 1000 m
Nodes Density (ρ)	3 $nodes/m^2$
Residual Energy Range (E_i)	0÷5 J
Maximum Transmission Radius (r)	1/(2$\sqrt{3}$) m
Transmission Rate (r_T)	1 kb/s
Number of run for each scenario	100

$T_{n-1} = T_{PL}$, where T_{PL} is the path-lifetime. This gives

$$\begin{aligned} \mathbf{v}_i &= \mathbf{v}_{i-1} + \sqrt{\frac{E_{i-1}}{P_{rec}T_{PL}}}\,\mathbf{u} = \\ &= \mathbf{v}_1 + \sum_{k=1}^{i-1}\sqrt{\frac{E_k}{P_{rec}T_{PL}}}\mathbf{u}, \quad i = 2, \ldots, n-1, \end{aligned}$$

where

$$\mathbf{u} = \frac{\mathbf{v}_n - \mathbf{v}_1}{\|\mathbf{v}_n - \mathbf{v}_1\|}$$

and T_{PL} can be found from

$$\mathbf{v}_n = \mathbf{v}_1 + \sum_{i=1}^{n-1}\sqrt{\frac{E_i}{P_{rec}T_{PL}}}\mathbf{u},$$

i.e.,

$$T_{PL} = \frac{1}{P_{rec}\,\|\mathbf{v}_n - \mathbf{v}_1\|^2}\left(\sum_{i=1}^{n-1}\sqrt{E_i}\right)^2.$$

The obtained positions $\mathbf{v}_i$ guarantee that the energy consumption is the minimum for each node in the data flow. Thus, nodes are closer or further from the following neighbour depending on their residual energies. In this way, mutual distances are determined and transmission powers will be consequently adapted. The solution found is the most energy-efficient placement of nodes, it requires a coordination centre (as the base station) for the collection of nodes' information and the computation of their positions.

TABLE II

SCENARIOS DESCRIPTION

Scenario			
#	Source (x,y)	Destination (x,y)	Length (m)
1	(0,800)	(200,1000)	$200\sqrt{2}$
2	(0,600)	(400,1000)	$400\sqrt{2}$
3	(0,400)	(600,1000)	$600\sqrt{2}$
4	(0,200)	(800,1000)	$800\sqrt{2}$
5	(0,0)	(1000,1000)	$1000\sqrt{2}$

III. RESULTS

Fig. 1 shows an example of a sensor field layout where nodes are stationary and deployed following a uniform distribution. In this section we present the results of the path-lifetime when we compare three different schemes of placement: random, evenly spaced and energy spaced. The routing issue is out of the scope of this paper, as we are only interested in the comparison of the placement of the nodes obtained from a random placement with the evenly spaced and the energy spaced solutions. We assume that a routing algorithm has established a path between source and destination. Since the optimal solution for the energy consumption requires nodes to lie on the direct route between source and destination [4], the natural choice for the comparison is that which selects the nodes placed at the shortest distance to the straight line. These nodes are indicated as full circles in Fig. 1. Both the second and the third placement schemes consider that nodes involved in the communication have all been placed on the straight line between source and destination. The second is the evenly spaced scheme, while the third is the energy spaced proposal. The positions for the second and the third schemes are represented in Fig. 1 with the symbol '+' and the full squares, respectively.

It should be noted that nodes' mobility could be a good solution for passing from a random placement to a sorted one in a distributed fashion and without the support of a coordination centre, but it is not considered here and it is left for future investigation.

Before discussing the results obtained, we focus on the details of the environment we considered. As in [6], we considered a static network of N nodes distributed according to a uniform spatial process in a square area A of side L, where $A = L^2$. The nodes' density is calculated as $\rho = (N\pi R^2)/A$, where R is the whole network radius. Evaluation parameters are summarized in Table I and it is important to note here that r is the maximum transmission radius allowed for a node in order to have the next hop in the path inside its transmission range, hence all the placement strategies will not permit a distance between each couple of nodes larger than r.

We tested the three schemes on five data flows which have different sources, destinations and path lengths, as characterized in Table II and shown in Fig. 1. When a node in the route depletes its battery, the whole path needs to be reconstructed or, at least, the exhausted node has to be replaced, otherwise the data flow suffers from wasteful disconnections. For these reasons, we considered the path-lifetime as the value of the minimum time duration that a node can be active in the current

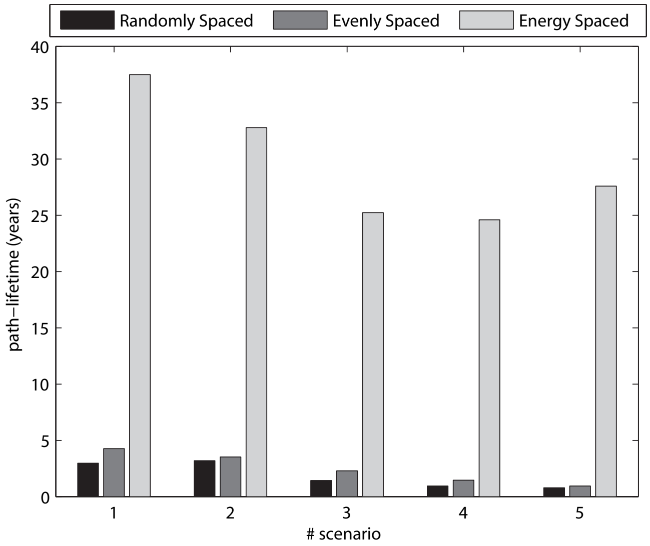

Fig. 2. Path Lifetime for the three schemes.

data flow, until its battery is completely out of energy.

In Fig. 2 the expected path-lifetime (evaluated through Monte Carlo simulations, with a confidence interval of 95%) is shown for the three schemes.

First, we observe that the evenly spaced placement achieves small improvements with respect to the configuration of nodes deployed randomly. This is a direct consequence of the results obtained in [4], because this approach does not consider the residual energy of the nodes.

Then, when we examine the comparative results of energy spaced and the other schemes, we notice that the placement, based on nodes residual energies, introduces dramatic positive effects on the path lifetime. Furthermore, in the energy spaced scheme, all the nodes in the path are characterized by the same lifetime. This is a very interesting property, because in many sensors applications, it is desirable to replace a large group of sensors instead of a single one.

Finally, a useful observation is that, in the environment we investigated, the results do not highlight any specific correlation between lifetime and length of the paths.

From Fig. 1 we can see that the longer the path, the higher the number of requested nodes between source and destination. Even if the number of nodes involved is larger, they are homogeneous in terms of residual energy, which is chosen uniformly in the same range $(0 \div 5J)$, and their transmission radius depends only on the density of nodes in the network, which is fixed in our performance evaluation.

IV. CONCLUSION

In this work we formulated an analytical model for the placement of nodes in a wireless sensors network, based on the residual energy of the nodes. The model allows us to find the most energy efficient positions in order to prolong the path lifetime of the nodes of a single data flow. The same idea can be applied for a real sensor network when the positions of the terminal nodes of a larger number of simultaneous flows are fixed and known from the deployment phase, thus improving the overall energy performance and increasing the lifetime of the whole network. The exact placement of the nodes can be an expensive operation, but the results obtained with our model are so remarkable that a question arises: “Can it be relevant for some sensors application to place the nodes in precomputed positions or is still better to deploy them randomly?”

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