Pulse Shape Optimization in Dispersion-Limited Direct Detection Optical Fiber Links

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***Abstract*—In this letter, we study the theoretical limits of optical communication channels affected by chromatic dispersion. By using as a metric the energy transfer ratio, we find the optimal transmitted pulse shape that allows minimizing the impact of dispersion. We show that these optimized pulses perform significantly better than standard nonreturn-to-zero/on-off keying (NRZ-OOK) modulation.**

***Index Terms*—Chromatic dispersion, energy transfer ratio, intersymbol interference (ISI), optical fiber communication.**

I. Introduction

RECENT work has pointed out a significant interest toward the development of optical transmission systems capable of tolerating a high degree of accumulated chromatic dispersion without requiring optical dispersion compensation [1]–[3]. In this scenario, most of the proposals are focused on finding dispersion-resilient modulation formats that can be implemented by (reasonably) simple modifications of the transmitter structure, while the receiver is usually unchanged, being still based on standard direct-detection. Some of the most successful techniques in reducing the amount of Intersymbol interference (ISI) generated by dispersion are:

- generation of an input pulse (for a single digital "1") with a proper amplitude and frequency modulation (or "chirp") [2];
- use of suitable line coding, such as duobinary/PSBT [1] or more complex codes [3].

Anyway, previous work has never focused on finding the ultimate *theoretical limits* of the optical dispersion channel (with direct detection). In this paper, we focus on these limits. In particular, we find the optimal transmitted pulse shape (in amplitude and phase) under the following assumptions and/or constraints:

- modulation is supposed to be, as in most of today optical systems, a memoryless binary on-off keying (OOK), without any line coding;
- transmitted pulse for a logical "1" has a time duration T strictly limited to the inverse of the bit rate R;
- optical transmitter is able to generate an arbitrary pulse shape (in amplitude and phase); a practical implementation can be based on an optical amplitude modulator followed by a phase modulator synchronously driven by two control signals;
- we consider a maximum energy transfer criterion over the dispersive channel, as discussed in Section II.

The goal of this letter is twofold. On one side, we find, for a given amount of dispersion, an "optimal" transmitter pulse shape. On the other side, we find under the above assumptions a theoretical limit to the maximum amount of accumulated chromatic dispersion that an OOK transmission system can tolerate before ISI becomes unacceptable. This limit can be taken as a reference to compare how close a given practical system is to the theoretically "optimal" one.

II. Mathematical Background

Being interested in dispersion-limited systems, we focus on a fiber transmission model which includes first order chromatic dispersion only, neglecting all other transmission impairments. Thus, we consider the following fiber (linear) transfer function:

$$H_F(f) = e^{-j\frac{\beta_2}{2}L(2\pi f)^2} = e^{j\gamma(\pi f T)^2} \tag{1}$$

where β_2 is the chromatic dispersion parameter and L is the fiber length. In order to simplify the expressions, we use the "Normalized Dispersion Index" (NDI), defined as [3]

$$\gamma = -2\beta_2 L R^2 \tag{2}$$

which enables to normalize the results to the system bit rate. Using this notation, the impulse response of the fiber is given by [3]

$$h_F(t) = \frac{e^{\frac{j\pi}{4}\mathrm{sign}(\gamma)}}{T\sqrt{\pi|\gamma|}} e^{-j\frac{\left(\frac{t}{T}\right)^2}{\gamma}} \tag{3}$$

We indicate the (complex envelope) of the transmitted optical field at the fiber input as the optical pulse $s_{in}(t)$. The resulting pulse at the fiber output is given by

$$s_{out}(t) = s_{in}(t) * h_F(t) \tag{4}$$

We define the input and output signal energies *over a bit duration* T as

$$\mathcal{E}_{in} = \int_{\frac{-T}{2}}^{\frac{T}{2}} |s_{in}(t)|^2\, dt \quad \mathcal{E}_{out} = \int_{\frac{-T}{2}}^{\frac{T}{2}} |s_{out}(t)|^2\, dt. \tag{5}$$

Following [5], we chose as our target the maximization over $s_{in}(t)$ of the "energy transfer ratio" (ETR), defined as

$$\mathrm{ETR} = \frac{\mathcal{E}_{\rm out}}{\mathcal{E}_{in}} \tag{6}$$

Basically, this means that we are looking for the input pulse $s_{in}(t)$ that allows, for a given channel, the maximum transfer

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of energy between input and output time windows. The concentration of the output pulse energy over a T time window is effective in both minimizing ISI and increasing the signal-to-noise ratio at the decision instant [4], [5]. The direct relation between the pulse energy concentration and ISI will be subsequently checked in Section III, in a detailed simulation including a realistic optical receiver.

The optimization problem we are facing is a canonical problem related to the optimization of a quadratic functional in $s_{in}(t)$ with a quadratic constraint [4]. The solution may be computed from the following homogeneous Fredholm integral equation of the second kind:

$$\int_{\frac{-T}{2}}^{\frac{T}{2}} \mathcal{K}(u,v)s_{in}(v)dv = \mu s_{in}(u) \tag{7}$$

where the kernel of the integral equation is

$$\mathcal{K}(u,v) = \int_{\frac{-T}{2}}^{\frac{T}{2}} h_F(z-u)h_F^*(z-v)dz \tag{8}$$

and where the *maximum eigenvalue* μ in (7) is exactly equal to the ETR defined in (6).

This problem has been solved in the literature for several types of low-pass filters [4]–[6]. In this paper, we solve it (for the first time to our knowledge) considering the fiber dispersive transfer function in (2) as the band-limiting filter. In this case, after some straightforward calculations, the kernel can be expressed as

$$\mathcal{K}(u,v) = \frac{\sin\left(\frac{u-v}{T\gamma}\right)}{\pi(u-v)} \cdot \exp\left[-j\frac{(u^2-v^2)}{T^2\gamma}\right] \tag{9}$$

The resulting integral equation can be solved by a numerical method described for example in [5, Sect. IV-A], which reduces the integral equation to a simple (and numerically stable) discrete eigenvalue problem.

III. Results

In Fig. 1 we show the resulting optimal pulses (in amplitude and phase) for increasing NDI γ (ranging from 0.1 to 0.3 in 0.05 steps). The signal amplitudes show a Gaussian-like return-to-zero (RZ) shape for low values of γ, while tend to become non-return-to-zero (NRZ) (i.e., constant over a bit duration) for high values of γ. Interestingly, this behavior is similar to the results obtained for any standard baseband low-pass filter, such as the single pole *RC*-filter studied in [4, Sect. 6.6], even though the channel transfer function (2) considered in this paper is very different and does not show, at first glance, a low-pass behavior. Moreover, the optimal signal phases (which show a parabolic behavior) allow to determine the optimal chirp that should be generated at the transmitter for a given value of γ.

Fig. 2 presents the main result of our paper. It shows the ETR as a function of γ for the optimized pulses, comparing it with standard chirpless NRZ (raised-cosine) and RZ (Gaussian, with full width at half maximum equal to $T/3$). We notice that the optimized pulses give significant advantage over standard NRZ

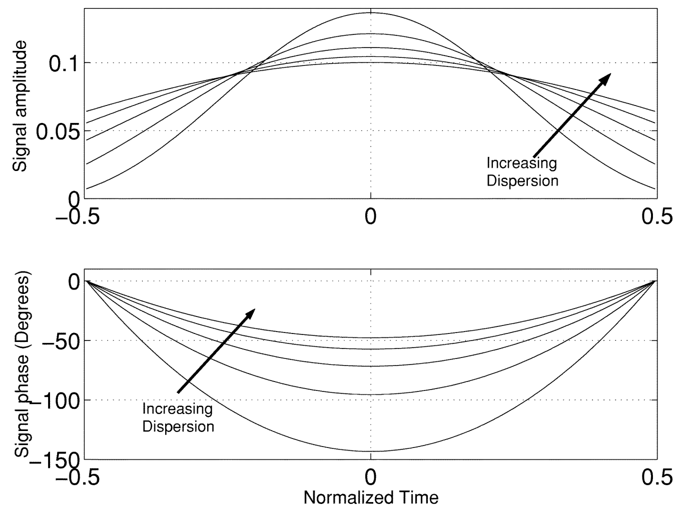

Fig. 1. Optimal pulse shape (amplitude and phase) for γ values ranging from 0.1 to 0.3 in 0.05 steps.

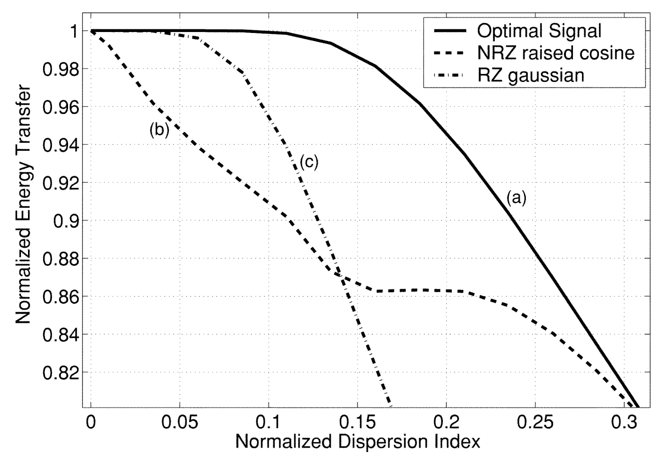

Fig. 2. Energy transfer ratio (ETR) versus γ for: (a) optimized signals; (b) raised cosine NRZ; and (c) Gaussian RZ.

or RZ transmission. For instance, if we fix the minimum acceptable ETR to 0.9, γ should be below 0.11 for NRZ, and 0.23 for optimized signals, meaning that under this assumption the maximum dispersion distance is doubled for the optimized signals with respect to NRZ (similar values hold true for Gaussian RZ). On one side, this graph determines for a given γ the maximum achievable ETR, and thus an upper bound on the transmission capabilities of the dispersion limited optical channel. On the other side, it allows to estimate the potential gain that can be achieved using optimized signals instead of standard chirpless RZ and NRZ signals.

In order to give a more practical insight on these results, and to connect the somehow theoretical performance parameter ETR to the power penalty due to dispersion, we performed a set of numerical simulations on a typical optical transmission system based on [7].

- *Transmitter Side:* Modulation at 10 Gbit/s using either optimal pulses or NRZ modulation format, with a (time-domain) raised-cosine shape, roll-off equal to 0.3 and no phase modulation (chirpless);
- *Link:* Only dispersion has been considered, as in (2); results will be expressed as a function of the accu-

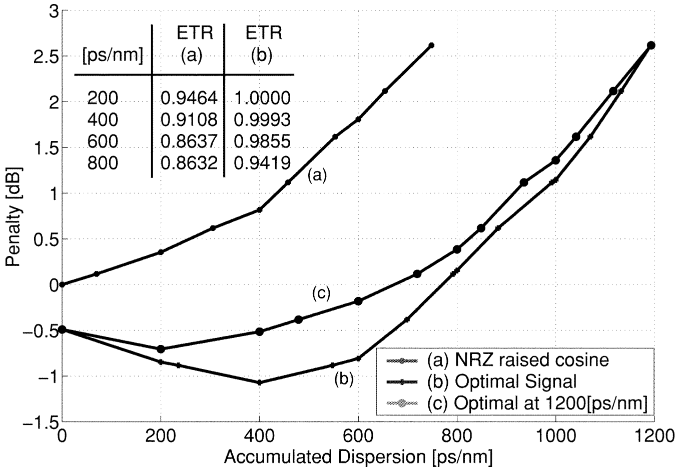

Fig. 3. Power penalty (dB) versus accumulated dispersion $D \cdot L$ [ps/nm] for systems using: (a) raised cosine NRZ; (b) optimized pulses (at each dispersion value); (c) optimized at 1200 [ps/nm] only.

mulated dispersion $D \cdot L$ measured in [ps/nm], where $D = -2\pi\beta_2 c/\lambda^2$ is the standard dispersion parameter that can be found in fiber datasheets;

- *Receiver side:* We considered amplified spontaneous emission (ASE) noise and a standard direct detection receiver using a Gaussian optical filter with 3-dB bandwidth equal to 25 GHz, a photodiode and an electrical fourth-order Bessel filter with 3-dB bandwidth equal to 7.5 GHz (corresponding to a typical SONET-SDH OC-192 receiver [8]).

Considering this scenario, we compare the performance of NRZ modulation with those of the optimal pulses derived in Section II. We set the reference optical signal-to-noise ratio (OSNR) at the receiver to 15.3 dB over a 0.1-nm bandwidth, giving rise, for NRZ without dispersion, to a *Q*-factor equal to 17 dB, resulting in a bit error probability below 10^{-12}.

In Fig. 3, we plot the power penalty as a function of the accumulated dispersion. The penalty is defined with respect to the NRZ system back-to-back sensitivity (i.e. without dispersion). Besides the reference NRZ curve, we plot the performance of a system with pulses optimized for *each* dispersion value, and the performance of a system that uses for *all* dispersion values a pulse optimized for $D \cdot L = 1200$ ps/nm. Some values of the corresponding ETRs are also reported on the figure. We note that:

- if we define, as it is commonly done, the dispersion limit at the point giving rise to a 2-dB power penalty, we note that for the NRZ system this limit is around 650 ps/nm accumulated dispersion, while for the optimized pulses it is around 1100 ps/nm; this shows the effectiveness of the optimized pulse, which allows to nearly double the dispersion limit and confirms that the theoretical results presented in Fig. 2 are indeed significant in practice;
- optimized pulses perform better than NRZ even for low dispersion values, where they show a *negative* penalty with respect to NRZ; this result can be explained by observing that the optimized pulse in this case (relatively low γ) tends to have an RZ Gaussian-like shape; RZ has been shown to give an advantage over NRZ for standard optical receivers with nonmatched optical filters [7];
- signal optimized for $D \cdot L = 1200$ ps/nm performs very well even for lower dispersion, and has only a small penalty with respect to the performance of the system optimized for each dispersion value, thus showing the robustness of the proposed optimized signals over all the dispersion range below the value at which they have been optimized;
- negative penalty around 400 ps/nm for the optimal pulses can be explained by the fact that for these dispersion values the pulses are better matched to the specific receiver filter.

IV. Discussion and Conclusion

In this letter, we presented an assessment of the ultimate theoretical limitation of the dispersion-limited optical channel with memoryless modulation, by finding the optimal pulse shape that allows minimizing the impact of dispersion at the receiver. Future works on this topic will extend our results to more complex modulations with memory and/or line coding (such as the PSBT/duobinary modulation), that are recently gaining a lot of attention in the optical communication community.

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