Rectangular Algebraic Space-Time Block Codes

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Abstract— **In this paper we present a new construction of Algebraic Space-Time codes with a temporal code length larger than the number of transmit antennas. We call these codes "Rectangular Space-Time Block Codes" - RSTBC. The construction of the RSTBCs is entirely inspired by and based on the construction of Perfect codes [1] which are the best square STBCs known in the litterature.**

I. Introduction

In order to achieve very high spectral efficiencies over wireless channels, we need multiple antennas at both transmitter and receiver ends. To increase the data rates and the diversity order, we have to use Space-Time codes. These codes consist in a temporal and spatial multiplexing in order to provide transmit spatial diversity and coding gain without sacrificing the bandwidth.

Many STBCs exist in the litterature. We are interested in full-rate, full-diversity STBCs. In [2], a code for the 2 transmit antennas case was presented. This approach was generalized for any number of transmit antennas M in [3], [4]. In [5], the authors proposed non full rate and full rate STBCs constructed by using division algebras. A division algebra naturally yields to a structured set of invertible matrices that can be used to construct Linear Dispersion codes.

In our last papers [6], [1], we have presented a new family of STBCs, called Perfect codes having moreover non-vanishing determinant (allowing them to achieve the diversity-multiplexing gain tradeoff) and a good energy efficiency. Perfect codes only exist for a number of antennas equal to 2, 3, 4 and 6. For the two antennas case, there exists an infinite family, and the best one is the Golden code [6]. The perfect codes have better performances than the best previously known STBCs.

All these STBCs are square, *i.e.* the temporal code length is equal to the number of transmit antennas. STBCs with temporal code length larger than the number of transmit antennas also exist in the litterature. We can cite the layered STBC VBLAST in [7], DBLAST in [8], WST in [9] and also the asymmetric TAST codes in [3]. But unfortunately, all these codes don't have the full rate, and not necessary, the full diversity properties.

We propose in this paper, a new construction of rectangular STBCs. The construction is completely inspired and based on the perfect code construction. These codes inherit the perfect codes properties : full-rate, full diversity, energy efficiency and higher coding gain.

II. System Model and Notations

We are interested, here, in the coherent case where the receiver perfectly knows channel coefficients. Let M and N be the respective number of transmit and receive antennas, and T the temporal code length. The received signal is :

$$Y_{N\times T} = H_{N\times M}\cdot X_{M\times T} + W_{N\times T}$$

where X is the transmitted codeword taken from STBC, H is the channel response and W is the i.i.d Gaussian noise. The information symbols will be taken in $q-QAM$ or $q-HEX$ constellations.

III. Rectangular STBC construction

The rectangular STBC construction is based on the perfect code construction [6], [1]. The idea is to exploit the layered structure of the perfect code offered by the cyclic division algebra in order to define asymmetric layers. For a given number of transmit antennas M, we consider the parameters of the $M\times M$ perfect code : the cyclic division algebra $(K/L,\sigma,\gamma)$, the ideal I of the ring of integers of K. (L is the base field which is equal either to $\mathbb{Q}[i]$ or to $\mathbb{Q}[j]$, K the extension field, σ the generator of the Galois group of K and $\gamma\in L$ verifying some conditions). We give, below, two examples in which all codes parameters are given. In order to have full rate codes, we have to transmit $M\cdot T$ information symbols, *i.e.* M information symbols from each antenna, at each time. By respecting the circular structure of the codeword matrix rows, which leads to the layered structure of the code, a codeword of the rectangular STBC is :

$$X=\begin{bmatrix} x_1 & x_2 & \dots & x_T \\ \gamma x_T & x_1 & \dots & x_{T-1} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma x_{T-(M-2)} & \gamma x_{T-(M-3)} & \dots & x_{T-(M-1)} \end{bmatrix}$$

where components x_i, $i=1\dots T$ depend on the information symbols,

$$x_i=\sum_{i=1}^{M} s_{1,i}v_i$$

$(v_k)_{k=1\dots M}$ is the ideal basis. The information symbols are taken in q-QAM constellations for $M=2$, 4 and in q-HEX constellations for $M=3$, 6. The rectangular STBC have the same properties as the perfect code : full rate, full diversity and good energy efficiency.

IV. Rectangular STBC $N=2$ and $T>2$

To construct the $2\times T$ rectangular STBC we use the Golden code parameters. A codeword is :

$$X_{2\times T}=$$
$$\begin{bmatrix} \alpha(s_1+\theta s_2) & \alpha(s_3+\theta s_4) & \dots & \alpha(s_{2T-1}+\theta s_{2T}) \\ \gamma\bar{\alpha}(s_{2T-1}+\bar{\theta}s_{2T}) & \bar{\alpha}(s_1+\bar{\theta}s_2) & \dots & \bar{\alpha}(s_{2T-3}+\bar{\theta}s_{2T-2}) \end{bmatrix}$$

$$X_{3\times T} = \begin{bmatrix} \alpha(s_1+\theta s_2+\theta^2 s_3) & \alpha(s_4+\theta s_5+\theta^2 s_6) & \dots & \alpha(s_{2T-2}+\theta s_{2T-1}+\theta^2 s_{2T}) \\ \gamma\sigma(\alpha(s_{2T-2}+\theta s_{2T-1}+\theta^2 s_{2T})) & \sigma(\alpha(s_1+\theta s_2+\theta^2 s_3)) & \dots & \sigma(\alpha(s_{2T-5}+\theta s_{2T-4}+\theta^2 s_{2T-3})) \\ \gamma\sigma^2(\alpha(s_{2T-5}+\theta s_{2T-4}+\theta^2 s_{2T-3})) & \gamma\sigma^2(\alpha(s_{2T-2}+\theta s_{2T-1}+\theta^2 s_{2T})) & \dots & \sigma^2(\alpha(s_{2T-8}+\theta s_{2T-7}+\theta^2 s_{2T-6})) \end{bmatrix} \quad (1)$$

where $\theta = \frac{1+\sqrt{5}}{2}$, $\bar{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1+i-i\theta$, $\bar{\alpha} = 1+i-i\bar{\theta}$ and $\gamma = i$. The s_i, $i = 1\dots 2T$ are the information symbols taken in q-QAM constellations.

To illustrate the performances of the $2\times T$ rectangular STBCs we have simulated these codes in a complete transmission scheme using the Sphere-Decoder. In figure 1, we plot the performances of the Golden code and the rectangular STBC for $N = M = 2$ and $T = 3, 4, 5, 6$, using 4-QAM constellations. The rectangular STBCs have the same diversity order as the Golden code. For high SNRs, we observe the coding gain advantage of the rectangular STBCs.

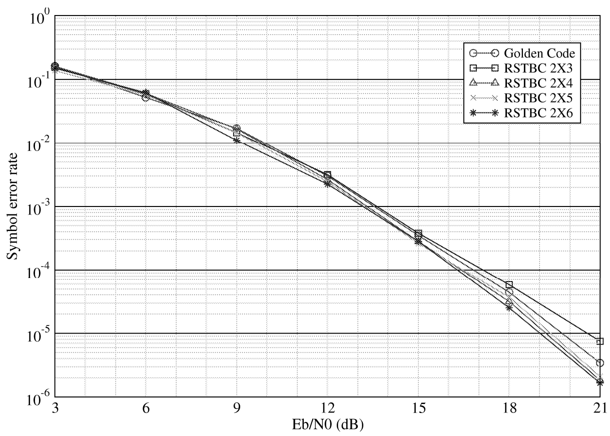

Fig. 1. Performances of rectangular STBC with $n_t = n_r = 2$, $T = 2, 3, 4, 5$and 6

V. Rectangular STBCs $M = 3$ and $T > 3$

Using the parameters of the 3×3 perfect code, we define the $3\times T$ rectangular STBCs. Codewords are given by equation (1), with $\theta = 2\cos\left(\frac{2\pi}{7}\right) = \zeta_7 + \zeta_7^{-1}$, $\alpha = 1+j+\theta$, $\sigma : \theta \mapsto \zeta_7^2 + \zeta_7^{-2}$ and $\gamma = j$. The s_i, $i = 1\dots 3T$ are the information symbols taken in q-HEX constellations (hexagonal constellation defined from the hexagonal lattice A_2).

In figure 2, we plot the symbol error rate as a function of the signal to noise ratio for the 3×3 perfect code and the rectangular STBCs with $N = M = 3$ and $T = 4$ and 5, using 4-HEX constellations. For high SNRs, we can observe the coding gain of the $3\times T$ rectangular STBCs compared to the 3×3 perfect code.

VI. Conclusions

We have presented full rate, full diversity, energy efficient rectangular space-time block codes offering good coding gains compared to the square STBCs. These codes are based on perfect code construction, and have an asymmetric layered structure. For a number of antennas for which a perfect code

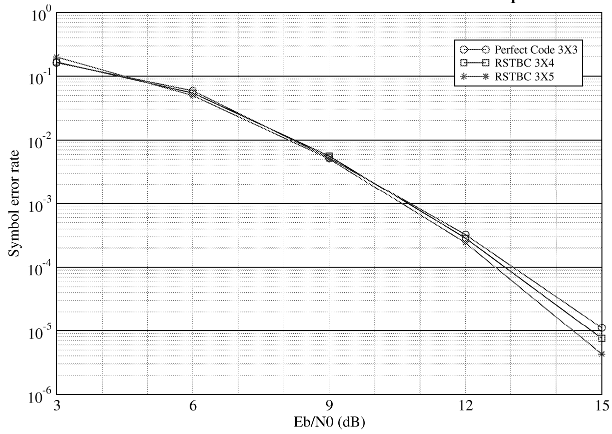

Fig. 2. Performances of rectangular STBC with $n_t = n_r = 3$, $T = 3$, 4 and 5

doesn't exist, rectangular STBCs can be constructed in the same way by using the TAST codes [3]. Rectangular STBCs can be interesting for systems constrained to have a small number of antennas.

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