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X-Codes: A low complexity full-rate high-diversity achieving precoder for TDD MIMO systems

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***Abstract*—We consider a time division duplex multiple-input multiple-output ($n_t \times n_r$ MIMO). Using channel state information (CSI) at the transmitter, singular value decomposition (SVD) of the channel matrix is performed. This transforms the MIMO channel into parallel subchannels, but has a low overall diversity order. Hence, we propose X-Codes which achieve a higher diversity order by pairing the subchannels, prior to SVD precoding. In particular, each pair of information symbols is encoded by a fixed 2×2 real rotation matrix. X-Codes can be decoded using n_r very low complexity two-dimensional real sphere decoders. Error probability analysis for X-Codes enables us to choose the optimal pairing and the optimal rotation angle for each pair. Finally, we show that our new scheme outperforms other low complexity precoding schemes.**

I. Introduction

In time division duplex (TDD) MIMO systems, where channel state information (CSI) is fully available at the transmitter, precoding techniques can provide large performance improvements.

Despite having low decoding complexity, the linear precoding schemes [1], [2], [11], [12] and the Tomlinson-Harashima precoder (THP) [3], [4], [5] have a diversity order of only one. This can be improved by transmitting over a subset of all possible modes of transmission, which results either in a rate loss or loss in power efficiency (since higher order modulation would have to be used to achieve the target spectral efficiency) [11], [12]. Other precoding techniques based on vector perturbation [6] and lattice reduction [7] can achieve good diversity but at much higher complexities.

In this paper, we consider singular value decomposition (SVD) of the channel, so that the MIMO channel can be seen as parallel subchannels [1], [2]. Unfortunately, such SVD precoded MIMO systems have no diversity gain. This fact motivates us to introduce X-Codes, which code across the subchannels prior to SVD precoding. The proposed X-Codes pair the subchannels with low diversity orders with those having high diversity orders. The X-Codes are named in such a way due to the structure of their encoding matrix. We show analytically that the diversity order of the system can be significantly improved. The pairing of subchannels is achieved by jointly coding the two subchannels with a two-dimensional real orthogonal matrix (which is effectively parametrized by a single angle). These angles are chosen *a priori* and do not change with each realization of the channel. This is the reason for using the term "Code" instead of "Precoder", since the optimal angles are fixed *a priori*. At the receiver, we show that the maximum likelihood (ML) decoding can be easily accomplished by using n_r simple two-dimensional real sphere decoders (SDs).

A precoding scheme that pairs subchannels to improve diversity, known as E-dmin, has been proposed recently in [10]. However, E-dmin is only optimized for 4-QAM modulation symbols and exhibits poor error performance, when used with higher order QAM modulation. In Section VI, we compare the error performance of the proposed X-Codes with E-dmin. We observe that for higher spectral efficiencies X-Codes have a superior error performance compared to E-dmin. In addition to this, the decoding complexity of X-Codes is lower than that of E-dmin. In particular, X-Codes can be decoded with n_r 2-dimensional real sphere decoders, whereas E-dmin requires $\frac{n_r}{2}$ 4-dimensional real sphere decoders.

II. System model

We consider a TDD system with $n_t \times n_r$ MIMO ($n_r \leq n_t$), where the channel state information (CSI) is known perfectly at both the transmitter and receiver. Let $\mathbf{x} = (x_1, \ldots, x_{n_t})^T$ be the vector of symbols transmitted by the n_t transmit antennas, where $(\cdot)^T$ denotes transposition, and let $\mathbf{H} = (h_{ij})$, $i = 1, \ldots, n_r$, $j = 1, \ldots, n_t$, be the $n_r \times n_t$ channel coefficient matrix, with h_{ij} as the complex channel gain between the j-th transmit antenna and the i-th receive antenna. The standard Rayleigh flat fading model is assumed with $h_{ij} \sim \mathcal{N}_c(0, 1)$, i.e., i.i.d. complex Gaussian random variables with zero mean and unit variance. The received vector with n_r symbols is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \tag{1}$$

where $\mathbf{z}$ is a spatially uncorrelated Gaussian noise vector such that $\mathbb{E}[\mathbf{z}\mathbf{z}^\dagger] = N_0 \mathbf{I}_{n_r}$, where † denotes the Hermitian transpose and $\mathbb{E}[\cdot]$ is the expectation operator. Such a system has a maximum multiplexing gain of n_r. Let the number of information symbols transmitted be n_s ($n_s \leq n_r$). Let $\mathbf{T}$ be the $n_t \times n_s$ precoding matrix which is applied to the information vector $\mathbf{u} = (u_1, \ldots, u_{n_s})^T$ to yield the transmitted vector

$$\mathbf{x} = \mathbf{T}\mathbf{u} \tag{2}$$

In general $\mathbf{T}$ is derived from the perfect knowledge of $\mathbf{H}$ at the transmitter. The way in which $\mathbf{T}$ is derived from $\mathbf{H}$ makes the various precoding schemes different from each

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other w.r.t. performance and encoding/decoding complexity. The transmission power constraint is given by

$$\mathbb{E}[\|\mathbf{x}\|^2] = P_T \quad (3)$$

where $\|\cdot\|$ denotes the Euclidean norm. Finally, we define the signal-to-noise ratio as $\gamma \triangleq \frac{P_T}{N_0}$.

III. SVD PRECODING AND X-CODES

SVD precoding is based on the singular value decomposition of the channel matrix $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}$ ($\mathbf{U} \in \mathbb{C}^{n_r \times n_r}$, $\mathbf{\Lambda} \in \mathbb{C}^{n_r \times n_r}$ and $\mathbf{V} \in \mathbb{C}^{n_r \times n_t}$), where $\mathbf{U}\mathbf{U}^\dagger = \mathbf{I}_{n_r}$, $\mathbf{V}\mathbf{V}^\dagger = \mathbf{I}_{n_r}$ and $\mathbf{I}_{n_r}$ denotes the $n_r \times n_r$ identity matrix. The diagonal matrix $\mathbf{\Lambda}$ contains the singular values λ_i ($i = 1, \ldots n_r$) of $\mathbf{H}$ in decreasing order ($\lambda_1 \geq \lambda_2 \cdots \geq \lambda_{n_r} \geq 0$). Let $\tilde{\mathbf{V}} \in \mathbb{C}^{n_s \times n_t}$ be the submatrix with the first n_s rows of $\mathbf{V}$. The precoder uses

$$\mathbf{T} = \tilde{\mathbf{V}}^\dagger \quad (4)$$

and the receiver gets

$$\mathbf{y} = \mathbf{H}\mathbf{T}\mathbf{u} + \mathbf{z} \quad (5)$$

Let $\tilde{\mathbf{U}} \in \mathbb{C}^{n_r \times n_s}$ be the submatrix with the first n_s columns of $\mathbf{U}$. The receiver then computes

$$\mathbf{r} = \tilde{\mathbf{U}}^\dagger \mathbf{y} = \tilde{\mathbf{\Lambda}}\mathbf{u} + \mathbf{w} \quad (6)$$

where $\mathbf{w} \in \mathbb{C}^{n_s}$ is still an uncorrelated Gaussian noise vector ($\mathbb{E}[\mathbf{w}\mathbf{w}^\dagger] = N_0\mathbf{I}_{n_s}$). $\tilde{\mathbf{\Lambda}} \triangleq \text{diag}(\lambda_1, \lambda_2, \cdots \lambda_{n_s})$, and $\mathbf{r} = (r_1, \ldots, r_{n_s})^T$. SVD precoding therefore transforms the channel into n_s parallel subchannels

$$r_i = \lambda_i u_i + w_i \qquad i = 1, \ldots, n_s \quad (7)$$

with non-negative fading coefficients λ_i. The overall error performance is dominated by the minimum singular value λ_{n_s}. In the special case of full-rate transmission ($n_s = n_r$), the resulting diversity order is only one. This problem is alleviated by the proposed X-Codes, where pairs of sub-channels are jointly coded.

We consider only the full-rate SVD precoding scheme with even n_r and $n_s = n_r$. It is straightforward to generate X-Codes with $n_s < n_r$ and odd n_s. Prior to SVD precoding, we now add a linear encoder $\mathbf{X} \in \mathbb{C}^{n_r \times n_r}$, which allows us to pair different subchannels in order to improve the diversity order of the system. The precoding matrix $\mathbf{T} \in \mathbb{C}^{n_t \times n_r}$ and the transmitted vector $\mathbf{x}$ are then given by

$$\mathbf{T} = \mathbf{V}^\dagger \mathbf{X}, \;\; \mathbf{x} = \mathbf{V}^\dagger \mathbf{X}\mathbf{u} \quad (8)$$

The code matrix $\mathbf{X}$ is determined by the list of pairings of the subchannels and the linear code generating matrix for each pair. Let the list of pairings be $\{(i_k, j_k), k = 1, 2 \cdots \frac{n_r}{2}\}$, where all i_k and j_k are distinct positive integers between 1 and n_r and $i_k < j_k$. On the k-th pair, consisting of subchannels i_k and j_k, the symbols u_{i_k} and u_{j_k} are jointly coded using a 2×2 matrix $\mathbf{A}_k$. In order to reduce the ML decoding complexity, we restrict the entries of $\mathbf{A}_k$ to be real valued. In order to avoid transmitter power enhancement, we impose an orthogonality constraint on each $\mathbf{A}_k$ and parameterize it with the angle θ_k.

$$\mathbf{A}_k = \begin{bmatrix} \cos(\theta_k) & \sin(\theta_k) \\ -\sin(\theta_k) & \cos(\theta_k) \end{bmatrix} \quad k = 1, \ldots n_r/2 \quad (9)$$

Each $\mathbf{A}_k$ is a 2×2 submatrix of the code matrix $\mathbf{X}$ as shown below.

$$\begin{aligned} X_{i_k,i_k} &= \cos(\theta_k) \quad & X_{i_k,j_k} &= \sin(\theta_k) \\ X_{j_k,i_k} &= -\sin(\theta_k) \quad & X_{j_k,j_k} &= \cos(\theta_k) \end{aligned} \quad (10)$$

where $X_{i,j}$ is the entry of $\mathbf{X}$ in the i-th row and j-th column. The orthogonality constraint on each $\mathbf{A}_k$ therefore implies that $\mathbf{X}$ is also orthogonal. We shall see later, that an optimal pairing in terms of achieving the best diversity order is one in which the k-th subchannel is paired with the $(n_r - k + 1)$-th subchannel. The code matrix $\mathbf{X}$ for this pairing has a cross-form structure and thus the name "X-Codes".

Each symbol in $\mathbf{u}$ takes values from a regular M^2-QAM constellation, which consists of two quadrature M-PAM constellations $\mathcal{S} \triangleq \{\beta(2i - (M-1)) \,|i = 0, 1, \cdots (M-1)\}$, where $\beta \triangleq \sqrt{\frac{3E_s}{2(M^2-1)}}$ and $E_s = \frac{P_T}{n_r}$ is the average symbol energy for each information symbol. Gray mapping is used to map the bits separately to the real and imaginary component of the symbols in $\mathbf{u}$.

IV. DECODING OF X-CODES

Given the received vector $\mathbf{y}$, the receiver computes $\mathbf{r} = \mathbf{U}^\dagger\mathbf{y}$. Using (1) and (8), we can write

$$\mathbf{r} = \mathbf{\Lambda}\mathbf{X}\mathbf{u} + \mathbf{w} \; = \mathbf{M}\mathbf{u} + \mathbf{w} \quad (11)$$

where $\mathbf{M} \triangleq \mathbf{\Lambda}\mathbf{X}$ is the equivalent channel gain matrix and $\mathbf{w} \triangleq \mathbf{U}^\dagger\mathbf{z}$ is a noise vector with the same statistics as $\mathbf{z}$.

Further let $\mathbf{r}_k \triangleq [r_{i_k}, r_{j_k}]^T$, $\mathbf{u}_k \triangleq [u_{i_k}, u_{j_k}]^T$, $\mathbf{w}_k \triangleq [w_{i_k}, w_{j_k}]^T$, for $k = 1, 2, \cdots n_r/2$. For each $k \in \{1, 2, \cdots \frac{n_r}{2}\}$, let $\mathbf{M}_k \in \mathbb{R}^{2\times 2}$ denote the 2×2 submatrix of $\mathbf{M}$ consisting of entries in the i_k and j_k rows and columns. Using (10) and the definition of $\mathbf{M}$ we have

$$\mathbf{M}_k = \begin{bmatrix} \lambda_{i_k}\cos(\theta_k) & \lambda_{i_k}\sin(\theta_k) \\ -\lambda_{j_k}\sin(\theta_k) & \lambda_{j_k}\cos(\theta_k) \end{bmatrix} \quad (12)$$

With these new definitions, (11) can be written as

$$\mathbf{r}_k = \mathbf{M}_k\mathbf{u}_k + \mathbf{w}_k, \;\; k \; = 1, 2, \cdots \frac{n_r}{2}. \quad (13)$$

From (13) it is clear that ML decoding of the transmitted information symbol vector $\mathbf{u}$ reduces to separate ML decoding of the k pairs. Note that $\mathbf{M}$ has real entries and the real and imaginary components of $\mathbf{w}$ are i.i.d. Furthermore, the real and imaginary components of $\mathbf{u}$ are also independent. Based on the above observations, it turns out that ML decoding for the k-th pair can be separated into independent ML decoding of the real and imaginary components of $\mathbf{u}_k$. Let $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of a complex argument, then the ML detection rule for the k-th pair is then given by

$$\Re(\hat{\mathbf{u}}_k) = \arg \min_{\Re(\mathbf{u}_k) \in \Re(\mathcal{S}^2)} \|\Re(\mathbf{r}_k) - \mathbf{M}_k\Re(\mathbf{u}_k)\|^2 \quad (14)$$

$$\Im(\hat{\mathbf{u}}_k) = \arg \min_{\Im(\mathbf{u}_k) \in \Im(\mathcal{S}^2)} \|\Im(\mathbf{r}_k) - \mathbf{M}_k \Im(\mathbf{u}_k)\|^2 \qquad (15)$$

where $\hat{\mathbf{u}}_k$ is the output of the ML detector for the k-th pair.

V. Performance evaluation and design of X-Codes

In this section, we analyze the word (block) error probability of X-Codes. Towards this end, we shall find the following Lemma useful (see [13] for a similar lemma and its proof).

Lemma 1: Given a real scalar channel modeled by $y = \sqrt{\alpha}x + n$, where $x = \pm\sqrt{E_s}$, $n \sim \mathcal{N}(0, \sigma^2)$, and the square fading coefficient α has $\mathbb{E}[\alpha] = 1$ and a cdf (Cumulative Density Function) $F(\alpha) = C\alpha^k + o(\alpha^k)$, for $\alpha \to 0^+$, where C is a constant and k is a positive integer[2], then the asymptotic error probability for $\gamma = E_s/\sigma^2 \to \infty$ is given by

$$P_e = \frac{C((2k-1).(2k-3) \ \cdots \ 5.3.1)}{2}\gamma^{-k} + o(\gamma^{-k})$$

■

Let P_k denote the word error probability for the k-th pair of subchannels, with the ML receiver as given by (14), (15). The overall word error probability for the transmitted information symbol vector is then given by, $P = 1 - \Pi_{k=1}^{\frac{n_r}{2}}(1 - P_k)$. Since the real and imaginary components of the channel have similar system model, we have $P_k = 1 - (1 - P_k^{'})^2$, where $P_k^{'}$ is the word error probability for the real component only. Let us further denote by $P_k^{'}(\Re(\mathbf{u}_k))$ the probability of the real part of the ML decoder decoding not in favor of $\Re(\mathbf{u}_k)$ when $\mathbf{u}_k$ is transmitted on the k-th pair. $P_k^{'}$ can then be expressed in terms of $P_k^{'}(\Re(\mathbf{u}_k))$ as follows

$$P_k^{'} = \frac{1}{M^2}\sum_{\Re(\mathbf{u}_k)} P_k^{'}(\Re(\mathbf{u}_k)) \qquad (16)$$

Getting an exact analytic expression is difficult, and therefore we resort to upper bounds for $P_k^{'}(\Re(\mathbf{u}_k))$. Using the union bound for $P_k^{'}(\Re(\mathbf{u}_k))$ and (16), we have

$$P_k^{'} \le \frac{1}{M^2}\sum_{\Re(\mathbf{u}_k)}\sum_{\Re(\mathbf{v}_k)\neq\Re(\mathbf{u}_k)} P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k)) \qquad (17)$$

where, $P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k))$ denotes the probability of the pairwise error event that, given $\mathbf{u}_k$ was transmitted on the k-th pair, the real part of the ML detector for the k-th pair decodes in favor of some other vector $\Re(\mathbf{v}_k)$. Due to Gaussian noise, we further have

$$P_k^{'} \le \frac{1}{M^2}\sum_{\Re(\mathbf{u}_k)}\sum_{\Re(\mathbf{v}_k)\neq\Re(\mathbf{u}_k)} \mathbb{E}\Big[Q\Big(\sqrt{\frac{d_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k)}{2N_0}}\Big)\Big] \qquad (18)$$

where $d_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k) \triangleq \|\mathbf{M}_k(\Re(\mathbf{u}_k) - \Re(\mathbf{v}_k))\|^2$, and $Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. The expectation in (18) is over the joint distribution of the channel gain $(\lambda_{i_k}, \lambda_{j_k})$. However in (18) evaluating the expectation over $(\lambda_{i_k}, \lambda_{j_k})$ is a difficult problem except for trivial cases (like $n_r = n_t = 2$). We therefore try to bound $d_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k)$ such that the bound depends only on λ_{i_k}. Since $\lambda_{i_k} \ge \lambda_{j_k} \ge 0$, using the definition of $\mathbf{M}$ and $\mathbf{M}_k$ we have

$$d_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k) \ge \lambda_{i_k}^2 \tilde{d}_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k) \qquad (19)$$

where $\tilde{d}_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k) \triangleq e_{k,1}^2$, $\mathbf{e_k} \triangleq \mathbf{A}_k(\Re(\mathbf{u}_k) - \Re(\mathbf{v}_k))$ and $e_{k,1}$ denotes the first component of the 2-dimensional vector $\mathbf{e_k}$. We further define the *generalized minimum distance* as follows :

$$g(\theta_k, M) = \min_{\Re(\mathbf{u}_k)\neq\Re(\mathbf{v}_k)} \tilde{d}_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k) \qquad (20)$$

Using the definition of $\mathbf{A}_k$ we have

$$\tilde{d}_k^2(\Re(\mathbf{u}_k), \Re(\mathbf{v}_k), \mathbf{A}_k) = \frac{6E_s}{(M^2-1)}(p\cos(\theta_k) + q\sin(\theta_k))^2 \qquad (21)$$

where the difference vector $\mathbf{z}_k = \Re(\mathbf{u}_k) - \Re(\mathbf{v}_k)$ can be written as $\sqrt{\frac{6E_s}{(M^2-1)}}(p, q)^T$, where $(p, q) \in \mathbb{S}_M$ and $\mathbb{S}_M \triangleq \{(p, q) | 0 \le |p| \le (M-1), 0 \le |q| \le (M-1), (p, q) \neq (0, 0)\}$. Using (21) in (20), we have

$$g(\theta_k, M) = \frac{6E_s}{(M^2-1)} \min_{(p,q)\in\mathbb{S}_M} (p\cos(\theta_k) + q\sin(\theta_k))^2 \qquad (22)$$

Since $Q(.)$ is a monotonically decreasing function with increasing argument, we can further upper bound (18) using (19) and (20) as follows :

$$P_k^{'} \le (M^2-1)\mathbb{E}\left[Q\left(\sqrt{\frac{3\gamma\lambda_{i_k}^2 g(\theta_k, M)}{n_r(M^2-1)}}\right)\right] \qquad (23)$$

The marginal pdf of the s-th eigenvalue λ_s^2 (for $\lambda_s^2 \to 0$) is given by [9]

$$f(\lambda_s^2) = C(s)(\lambda_s^2)^{N_t(s)N_r(s)-1} + o\Big((\lambda_s^2)^{N_t(s)N_r(s)-1}\Big) \qquad (24)$$

where $N_t(s) \triangleq (n_t - s + 1)$, $N_r(s) \triangleq (n_r - s + 1)$ and $C(s)$ is a constant given in [9]. Using the pdf in (24), the cdf $F_s(u) = P(\lambda_s^2 \le u)$ (for $u \to 0$) can be easily evaluated. Using Lemma 1, the bound in (23) can be further written as

$$P_k^{'} \le (M^2-1)b_k\Big(\frac{3\gamma g(\theta_k, M)}{n_r(M^2-1)}\Big)^{-\delta_k} + o(\gamma^{-\delta_k}) \qquad (25)$$

where $\delta_k \triangleq (n_t - i_k + 1)(n_r - i_k + 1)$ and

$$b_k \triangleq \frac{C(i_k)((2\,\delta_k - 1)\cdots 5 \cdot 3 \cdot 1)}{2\,\delta_k}$$

From (25) it is clear that the diversity order achievable by the k-th pair is at least δ_k. The diversity order achievable for the overall system (combined effect of all the pairs) is determined by the pair with the lowest diversity order. Let δ_{ord} denote the overall diversity order. Based on the above discussion δ_{ord} can be lower bounded as follows.

$$\delta_{ord} \ge \min_k \delta_k. \qquad (26)$$

For a given $n_t \times n_r$ MIMO, the design of optimal X-Codes depends upon the following two questions. *i)* What is the

[2]Any function $f(x)$ in a single variable x is said to be $o(g(x))$ i.e. $f(x) = o(g(x))$ if $\frac{f(x)}{g(x)} \to 0$ as $x \to 0$

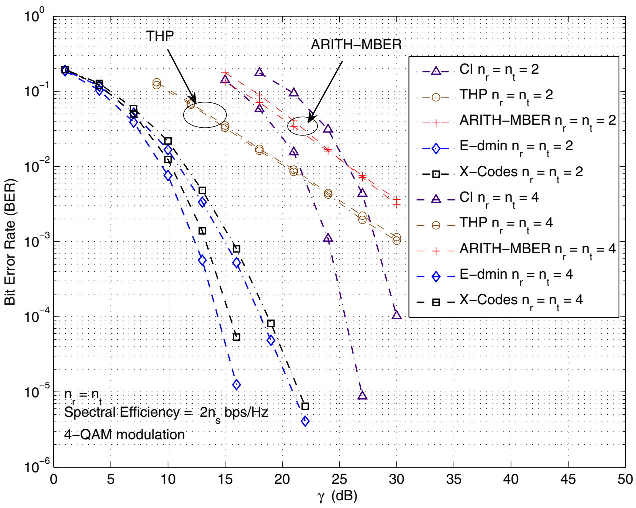

Fig. 1. Comparison with various precoders for $n_r = n_t = n_s = 2, 4$ and M = 2 (4-QAM) modulation. Target spectral efficiency is equal to $2n_s$ bps/Hz.

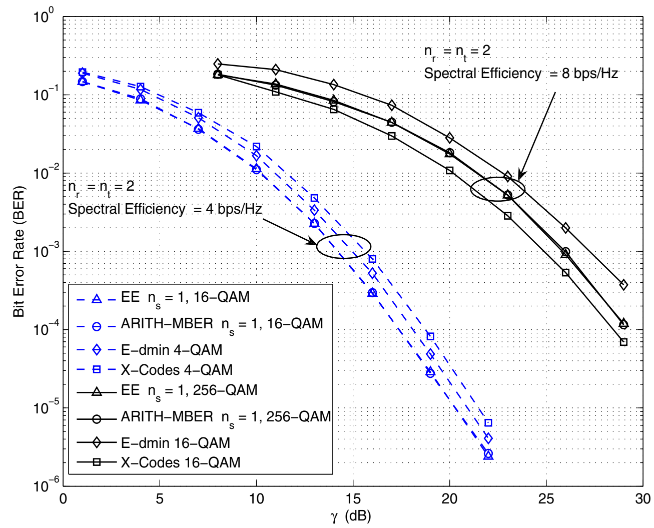

Fig. 2. Comparison between various precoders for $n_r = n_t = 2$ and target spectral efficiency = 4,8 bps/Hz.

optimal pairing of subchannels in terms of achieving the best diversity order? *ii*) What is the optimal angle for each pair? From the lower bound on δ_{ord} (given by (26)) it is clear that the following pairing of subchannels achieves the best lower bound

$$i_k = k\,,\; j_k = (n_r - k + 1),\; k\; = 1, 2 \cdots \frac{n_r}{2}. \tag{27}$$

Note that this corresponds to a cross-form generator matrix $\mathbf{X}$. Note that the pairing in (27), is not the only pairing which achieves the best lower bound. The lower bound on the overall diversity order is then given by,

$$\delta_{ord} \geq (\frac{n_r}{2} + 1)(n_t - \frac{n_r}{2} + 1) \tag{28}$$

In general, it can be shown that if only n_s (n_s even) out of the n_r subchannels are used for transmission, the lower bound on the achievable diversity order is $(n_r - \frac{n_s}{2} + 1)(n_t - \frac{n_s}{2} + 1)$.

Finding the optimal angle for the k-th pair is difficult, due to lack of exact error probability expressions. We therefore optimize error bounds, such as in (25). This is optimized by choosing the angle which maximizes $g(\theta_k, M)$.

VI. Simulation Results

In this section, we compare the performance of X-Codes with other precoders. The subchannel pairing for the X-Code is given by (27). The angle used for the subchannels is derived by maximizing $g(\theta_k, M)$ w.r.t. θ_k.

Comparisons are made with *i*) the E-dmin (equal dmin precoder proposed in [10]), *ii*) the Arithmetic mean BER precoder (ARITH-MBER) proposed in [11], *iii*) the Equal Energy linear precoder (EE) based upon optimizing the minimum eigenvalue for a given transmit power constraint [12]), *iv*) the TH precoder based upon the idea of Tomlinson-Harashima precoding applied in the MIMO context [5]) and *v*) the channel inversion (CI) known as Zero Forcing precoder.

Among all the considered schemes (except CI), we observe that E-dmin and X-Codes have the best diversity order of $(n_r - \frac{n_s}{2} + 1)(n_t - \frac{n_s}{2} + 1)$, when n_s ($n_s \leq n_r$) subchannels are used for transmission. The CI scheme achieves infinite diversity, but it suffers from power enhancement at the transmitter.

In Fig. 1, we plot the bit error rate (BER) of all precoders for $n_r = n_t = n_s = 2, 4$ and a target spectral efficiency of $2n_s$ bps/Hz (4-QAM). It is observed that E-dmin performs the best. X-Codes perform only 0.6 dB away from E-dmin (at BER = 10^{-3}). However, E-dmin has this performance gain at a higher encoding and decoding complexity. The performance of CI is inferior compared to X-codes and E-dmin. This is primarily due to enhanced transmit power requirement arising from the bad conditioning of the channel. THP is able to achieve only first order diversity. ARITH-MBER and EE precoder also achieve only a first order diversity (EE is not shown since it has a BER performance almost same as that of ARITH-MBER). Later we shall see that the performance of ARITH-MBER and EE is improved when only a subset of the subchannels are used. However we shall also observe that for higher spectral efficiencies, X-Codes outperform ARITH-MBER, EE and also the E-dmin precoder.

In Fig. 2, we plot the BER for $n_r = n_t = 2$, and a target spectral efficiency of 4,8 bps/Hz (4-, 16-QAM). It is observed that for a target spectral efficiency of 4 bps/Hz, the best performance is achieved by ARITH-MBER and EE using only n_s=1 subchannel with 16-QAM modulation. X-Codes with 4-QAM modulation performs the worst. X-codes perform about 1.2 dB worse (at BER = 10^{-3}) compared to ARITH-MBER and EE. For a target spectral efficiency of 8 bps/Hz the results are totally different. X-Codes with 16-QAM modulation performs the best, and E-dmin performs the worst. Also the performance of X-codes is better than that of ARITH-MBER/EE by about 0.8 dB (at BER = 10^{-3}). Similar observations are made for a $n_r = n_t = 4$ system in Fig. 3.

It can be observed from Figs. 2 and 3 that for higher spectral efficiencies X-Codes outperform other precoders.

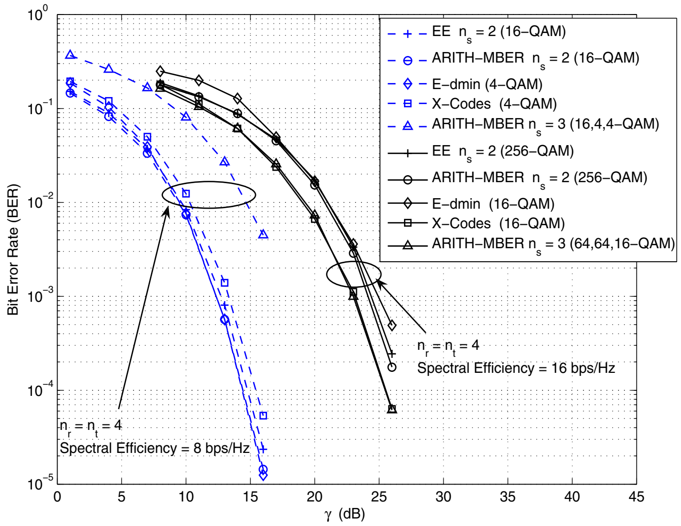

Fig. 3. Comparison between various precoders for $n_r = n_t = 4$ and target spectral efficiency = 8,16 bps/Hz.

VII. Complexity

In this section, we discuss the computational complexity of X-Codes and compare it with other precoding schemes. All precoders need to compute either the SVD, pseudo-inverse or QR decomposition of $\mathbf{H}$, each of which has a complexity of $O(n_r^3)$. These operations are computation intensive. However TDD is generally employed in a slowly fading channel, and therefore these computations can be performed once, and can be used until the channel changes. We, therefore, do not consider these decompositions in the discussion below.

A. Encoding Complexity

The encoding complexity of all the schemes is $O(n_r n_t)$, which is due to the transmit preprocessing filter. Apart from this, CI and X-Codes would have the lowest complexity. This is so because, linear precoders need to compute an extra preprocessing matrix (in addition to SVD). The THP also has to do successive interference pre-cancellation (in addition to QR). On the other hand, E-dmin and X-Codes need to only compute SVD, which automatically gives the pre-processing and the post-processing matrices. Also, X-Codes have lower encoding complexity compared to E-dmin, since unlike E-dmin the coding matrices $\mathbf{A}_k$ are fixed *a priori*. CI has an even lower complexity since there is no spatial coding.

B. Decoding Complexity

Due to the post-processing matrix filter, the decoding complexity of all the schemes have a square dependence on n_r. The linear precoders, CI and THP employ post processing at the receiver, which enables independent ML decoding for each subchannel. E-dmin and X-Codes on the other hand use sphere decoding to jointly decode pairs of subchannels. ML decoding for X-Codes is accomplished by using n_r 2-dimensional real sphere decoders. However E-dmin requires $\frac{n_r}{2}$ 4-dimensional real sphere decoders. The average complexity of sphere decoding is cubic in the number of dimensions (and is invariant w.r.t. modulation alphabet size M) [8], and therefore X-Codes have a much lower decoding complexity, when compared to E-dmin. This is attributed to the fact that the ML decoder for X-Codes separates into independent ML decoders for the real and imaginary components, which is not true for E-dmin.

VIII. Conclusion and Future Work

The proposed X-Codes are able to achieve full-rate and high diversity at a low complexity by pairing the subchannels prior to SVD precoding. Future work will also address the reduction in decoder complexity and the generation of soft outputs. Additional work will study X-Precoders, in which the optimal angle for each pair is chosen at both the transmitter and receiver every time the channel changes. The error performance of any other pairing, which also achieves the lower bound on δ_{ord}, is a subject of future work.

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