Performance of High Diversity Multidimensional Constellations

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In this paper we evaluate analytically the performance of component-interleaved multidimensional constellations over the Rayleigh fading channel. The concept of *modulation diversity* was introduced in [3] as a way to enhance the error performance of communication systems over the Rayleigh fading channel. Several multidimensional signal sets with high diversity order have been proposed to obtain substantial gains over conventional modulation schemes in a fading environment [2]. Modulation diversity can also be viewed as a special form of signal space coding producing highly efficient (in terms of bandwidth and power) schemes over the fading channel [3].

The modulation diversity order of a signal set is defined as the minimum number of *different* components between any two distinct points of the set. This definition applies to every modulation scheme and affects its performance over the fading channel in conjunction with component interleaving. By use of component interleaving, fading attenuation over different space dimensions become statistically independent. An attractive feature of these schemes is that we have an improvement of error performance without even requiring the use of conventional channel coding.

Most research work done in the area of multidimensional rotated constellation uses the Chernoff bound for performance analysis. This provides simple design criteria for good signal sets. However, the looseness of the Chernoff bound prevents to obtain precise performance results so that, in most cases, simulation is needed. A first step to improve the accuracy of error performance analysis has been made by the authors in [4] focusing on two-dimensional schemes. In this work we extend the analysis to multidimensional constellations and provide additional insight and some optimality criteria that can be helpful in the design for the Rayleigh fading channel.

The channel model is the following. Let $\mathbf{x} = (x_1, x_2, \ldots x_n)$ denote a transmitted signal vector from a given n-dimensional constellation S. Received signal samples are then given by $y_i = g_i x_i + n_i$ for $i = 1, 2, \ldots n$. Here, g_i are (real) independent Rayleigh-distributed random variables with unit second moment (i.e., $E[g_i^2] = 1$) representing the fading coefficients and n_i are real Gaussian random variables with mean zero and variance $N_0/2$ representing the additive noise. We also denote $\mathbf{g} = (g_1, g_2, \ldots g_n)$, $\mathbf{n} = (n_1, n_2, \ldots n_n)$, and $\mathbf{y} = (y_1, y_2, \ldots y_n)$ and write $\mathbf{y} = \mathbf{g} \odot \mathbf{x} + \mathbf{n}$. Note that when all the $g_i = 1$ we fall back to the well known Additive White Gaussian Noise (AWGN) channel.

For maximum-likelihood (ML) detection we compute the sample metrics

$$m(\mathbf{x}|\mathbf{y},\mathbf{g}) = ||\mathbf{y} - \mathbf{g} \odot \mathbf{x}||^2 \quad \forall \ \mathbf{x} \in S \tag{1}$$

where $||\cdot||^2$ is the standard Euclidean norm, and decide for the signal $\widehat{\mathbf{x}}$ attaining the minimum value of $m(\mathbf{x}|\mathbf{y},\mathbf{g})$.

It can be shown that the error probability of such a system is essentially dominated by the following parameters: i) the *modulation diversity* L, i.e., the minimum number of *different* components between any two distinct points in the signal set; ii) the *minimum* L*-product distance* $d_{\mathrm{p,min}}^{(L)} = \min_{\mathbf{x},\widehat{\mathbf{x}}} \prod_{x_i \neq \widehat{x}_i} |x_i - \widehat{x}_i|$ of the constellation. Good signal sets maximize L and $d_{\mathrm{p,min}}^{(L)}$ under an average energy constraint, namely, $E[||\mathbf{x}||^2] =$ constant. For $L \to \infty$ the error rate performance of a signal constellation over a fading channel approaches the one over AWGN channel.

A standard approach to error performance analysis consists of evaluating the symbol error probability of a signal set S by using the union bound

$$P(e) \leq \frac{1}{|S|} \sum_{\mathbf{x} \in S} \sum_{\mathbf{x} \neq \widehat{\mathbf{x}}} P(\mathbf{x} \to \widehat{\mathbf{x}}) \ . \tag{2}$$

Each PEP $P(\mathbf{x} \to \widehat{\mathbf{x}})$ is commonly approximated by using the Chernoff bound or other upper bound. However, as we show in the following theorem, the PEP can be calculated exactly over the Rayleigh fading channel with component interleaving.

Theorem. *Let us define* $\mathbf{x} = (x_1, x_2, \ldots x_n)$, $\widehat{\mathbf{x}} = (\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_n)$, $\delta_i = |x_i - \widehat{x}_i| \neq 0$ *for* $i = 1, 2, \ldots n$, *and let* $\mathbf{x} \to \widehat{\mathbf{x}}$ *represent a pairwise error event. Additionally, assume that all* δ_i*'s are distinct. The pairwise error probability is then given by*

$$P(\mathbf{x} \to \widehat{\mathbf{x}}) = \frac{1}{2} \sum_{i=1}^{n} \left(1 - \frac{\delta_i}{\sqrt{4N_0 + \delta_i^2}} \right) \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{\delta_i^2}{\delta_i^2 - \delta_j^2} \tag{3}$$

The asymptotic expansion (as $N_0 \to 0$) of this result lead to

$$P(\mathbf{x} \to \widehat{\mathbf{x}}) = \frac{\gamma_L}{[d_p^{(L)}]^2} N_0^L + O(N_0^{L+1}) \qquad N_0 \to 0 \tag{4}$$

where $d_p^{(L)} = \prod_{i=1}^{L} \delta_i$ and $\gamma_L = 2^{L-1}(2L-1)!!/L!$. and the first term is an upper bound because the series expansion is alternating in signs. This approximation improves on the corresponding Chernoff bound given in [1] by the factor $\gamma_L/4^L$.

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