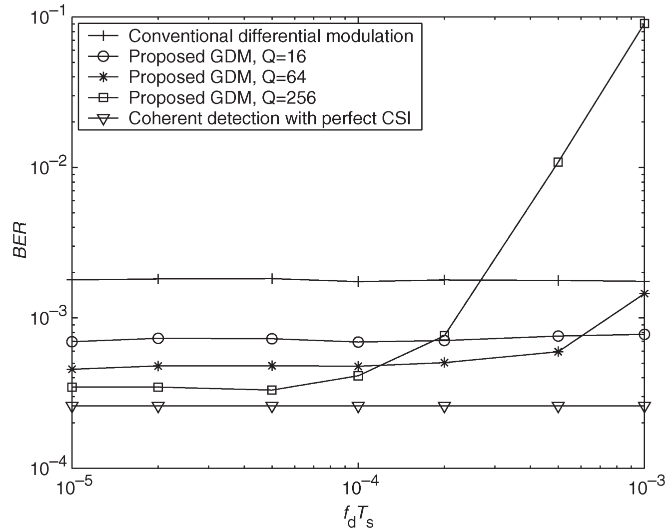

Fig. 4. BER performance of the proposed differential modulation scheme with different normalized Doppler frequencies, three relays, DQPSK modulation, and $E_b/N_0 = 15$ dB.

symbols (time-domain differential modulation) or on the adjacent subcarriers of the same OFDM symbol (frequency-domain differential modulation). Let $x_i(n)$, $n = 0, 1, \ldots, N-1$, be the modulated symbol on the nth subcarrier in the ith OFDM symbol at the source, where N is the number of subcarriers. For frequency-domain differential modulation, $x_i(0)$ is the initial reference symbol with an average power of $P_{s,1}$. All these N symbols in the ith OFDM symbol are divided into B blocks, and each block consists of Q symbols. Here, we assume that $N = BQ$. Similar to (2), the bth reference symbol in the ith OFDM symbol is differentially encoded as

$$x_i(bQ) = x_i\left((b-1)Q\right) S_i(bQ), \qquad b = 1, 2, \ldots, B-1 \quad (31)$$

where $S_i(bQ)$ is the information symbol. Likewise, the normal symbols are encoded as

$$x_i(bQ+q) = \sqrt{\frac{P_{s,2}}{P_{s,1}}} x_i(bQ) S_i(bQ+q), \qquad q = 1, 2, \ldots, Q-1. \quad (32)$$

The differential detection and power allocation can also be performed in a similar way as described in Sections IV and V.

VII. Conclusion

We proposed a GDM scheme for wireless relay networks with AF relaying. We investigated the optimal power allocation among the reference symbol and the normal symbols to maximize the output SNR. The proposed scheme with a block length of 256 provided a 2.5-dB performance improvement at a BER of 10^{-3} over the conventional scheme in slow Rayleigh fading channels. This differential modulation scheme can readily be extended to wireless relay networks with DF relaying or distributed space–time coding.

References

[1] D. Gesbert, M. Shafi, S. Da-shan, P. J. Smith, and A. Naguib, "From theory to practice: An overview of MIMO space–time coded wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 3, pp. 281–302, Apr. 2003.
[2] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.
[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[4] M. O. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.
[5] M. O. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963–1968, Nov. 2004.
[6] Q. Zhao and H. Li, "Differential modulation for cooperative wireless systems," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 2273–2283, May 2007.
[7] T. Himsoon, W. Su, and K. J. R. Liu, "Differential transmission for amplify-and-forward cooperative communications," *IEEE Signal Process. Lett.*, vol. 12, no. 9, pp. 597–600, Sep. 2005.
[8] T. Himsoon, W. Su, and K. J. R. Liu, "Differential modulation for multi-node amplify-and-forward wireless relay networks," in *Proc. IEEE WCNC*, Apr. 2006, vol. 2, pp. 1195–1200.
[9] W. Cho and L. Yang, "Resource allocation for amplify-and-forward relay networks with differential modulation," in *Proc. IEEE GLOBECOM*, Nov. 2007, pp. 1668–1672.
[10] W. Cho and L. Yang, "Joint energy and location optimization for relay networks with differential modulation," in *Proc. IEEE ICASSP*, Apr. 2007, pp. 153–156.
[11] L. Lampe, R. Schober, V. Pauli, and C. Windpassinger, "Multiple-symbol differential sphere decoding," *IEEE Trans. Commun.*, vol. 53, no. 12, pp. 1981–1985, Dec. 2005.
[12] R. Schober, W. H. Gerstacker, and J. B. Huber, "Decision-feedback differential detection of MDPSK for flat Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1025–1035, Jul. 1999.
[13] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 2000.
[14] R. H. Clarke, "A statistical theory of mobile-radio reception," *Bell Syst. Tech. J.*, vol. 47, no. 6, pp. 957–1000, Jul./Aug. 1968.

Algebraic Multiuser Space–Time Block Codes for a 2 × 2 MIMO

Yi Hong, *Member, IEEE*, and
Emanuele Viterbo, *Senior Member, IEEE*

***Abstract*—We focus on multiuser space–time block codes (STBCs) for a 2 × 2 multiple-input–multiple-output (MIMO) uplink transmission. Based on full-diversity algebraic rotations, we propose a family of multiuser STBCs constructed to minimize a truncated union-bound (UB) approximation. The proposed codes do not incur in any peak-to-average penalty and outperform all previously known multiuser STBCs.**

***Index Terms*—Full-diversity algebraic rotation, multiple-input multiple-output (MIMO), multiuser.**

Manuscript received July 23, 2008; revised November 10, 2008 and December 19, 2008. First published January 13, 2009; current version published May 29, 2009. This work was supported by STREP Project IST-026905 (MASCOT) within the Sixth Framework Program of the European Commission. This paper was presented in part at the International Symposium on Spread Spectrum Techniques and Applications (ISSSTA), Bologna, Italy, August 2008, and the IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Cannes, France, September 2008. The review of this paper was coordinated by Prof. E. Bonek.

Y. Hong is with Swansea University, Swansea, U.K. (e-mail: yi.winnie.hong@gmail.com).
E. Viterbo is with the Dipartimento di Elettronica Informatica e Sistemistica, Università della Calabria, 87036 Rende, Italy (e-mail: viterbo@deis.unical.it).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.
Digital Object Identifier 10.1109/TVT.2009.2012864

I. Introduction

Space–time block codes (STBCs) have intensively been studied for a *single-user* multiple-input multiple-output (MIMO). Recently, Gartner and Bolcskei [1] extended the idea of a single-user STBC to the *multiuser* case. An ad hoc example of a two-user 2 × 2 MIMO STBC was given based on an Alamouti structure. Motivated by the code design in [1], an algebraic construction of multiuser STBCs was presented in [2] to achieve the diversity–multiplexing tradeoff for users with a single transmit antenna ($n_t = 1$) and any number of receive antennas n_r.

Another family of multiuser STBCs was proposed in [3], where the design criteria were generalized for more than two users and based on pairwise error probability analysis. However, these codes incur in a large *peak-to-average penalty* since some elements in the codeword matrices are zero.

In our paper, we consider a 2 × 2 multiuser MIMO over quasi-static fading multiple-access channels (MACs). Unlike the multiuser codes in [1] and [3], we propose the code design criteria based on a truncated union-bound (UB) approximation. By exploiting the algebraic full-diversity rotations [4], we show how to construct a family of STBCs for any number of users to minimize the error probability of the truncated UB, without the peak-to-average penalty of [3]. Within this family, we present a code design example for a two-user 2 × 2 MIMO. Finally, we show by simulation that the proposed codes outperform previously known STBCs [1], [3].

Notations: Boldface letters are used for column vectors, whereas capital boldface letters are used for matrices. Superscripts ^{T} and † denote transposition and Hermitian transposition, respectively. Let $\mathbb{Q}$ and $\mathbb{C}$ denote the field of rational and complex numbers, respectively. The $\mathrm{vec}(\cdot)$ operator stacks the m column vectors of an $n \times m$ complex matrix into an mn column vector. Let $\|\cdot\|$ denote the Frobenius norm, and let $\mathbb{E}[\cdot]$ denote the mean of a random variable.

II. System Model

We consider a synchronous quasi-static fading MIMO MAC, where K users with n_t transmit antennas simultaneously communicate with a base station with n_r receive antennas. For the kth user, let

$$\mathbf{s}^{(k)} \triangleq \left[s_1^{(k)}, \dots, s_i^{(k)}, \dots, s_N^{(k)}\right]^T \in \mathbb{C}^N$$

be the information symbol vector of length N, where $s_i^{(k)}$, $i=1, \dots, N$, denote independent information symbols drawn from some complex quadratic-amplitude modulation (QAM) constellation.

The symbol vector $\mathbf{s}^{(k)}$ is encoded with the kth user STBC $\mathcal{C}_k$. This produces the kth *user codeword matrix* $\mathbf{C}_k \in \mathbb{C}^{n_t \times N}$ spanning over N channel uses, which is defined as

$$\mathbf{C}_k \triangleq \left[\mathbf{c}_1^{(k)}, \dots, \mathbf{c}_j^{(k)}, \dots, \mathbf{c}_{n_t}^{(k)}\right]^T \in \mathcal{C}_k$$

where $\mathbf{c}_j^{(k)} \triangleq [c_{j,1}^{(k)}, \dots, c_{j,N}^{(k)}]^T \in \mathbb{C}^N$, and $c_{j,n}^{(k)}$ denotes the space–time block-coded symbol transmitted at the jth transmit antenna of user k over the nth channel use. The aforementioned choice implies that the users transmit at a rate of one symbol per channel use.

All K users are assumed to simultaneously transmit their codeword matrices $\mathbf{C}_k$, yielding the following $n_t K \times N$ *joint codeword matrix*:

$$\mathbf{X} = \{x_{i,j}\} \triangleq \left[\mathbf{C}_1^T, \dots, \mathbf{C}_k^T, \dots, \mathbf{C}_K^T\right]^T \in \mathcal{C} \tag{1}$$

where $i = 1, \dots, n_t K$, $j = 1, \dots, N$, and $\mathcal{C}$ is the *joint codebook*. In this paper, we assume that each user employs a linear STBC [6, Definition 5], so that the elements $c_{j,n}^{(k)}$ are linear combinations of N complex QAM symbols.

At the receiver, the received signal matrix $\mathbf{Y} \in \mathbb{C}^{n_r \times N}$ can be written as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{2}$$

where $\mathbf{N} \in \mathbb{C}^{n_r \times N}$ is the complex white Gaussian noise with independent identically distributed (i.i.d.) entries $\sim\mathcal{N}_{\mathbb{C}}(0, N_0)$, and $\mathbf{H} \in \mathbb{C}^{n_r \times K n_t}$ is defined as

$$\mathbf{H} = \left[\mathbf{H}^{(1)}, \dots, \mathbf{H}^{(k)}, \dots, \mathbf{H}^{(K)}\right]$$

where $\mathbf{H}^{(k)} \triangleq \{H_{i,j}^{(k)}\} \in \mathbb{C}^{n_r \times n_t}$ denotes the channel matrix associated with the kth user, which is assumed to remain constant during the transmission of a codeword and to take on independent values from one codeword to another. The elements $H_{i,j}^{(k)}$ are the channel coefficients from the jth transmit antenna to the ith receive antenna for user k and assumed to be i.i.d. circularly symmetric Gaussian $\sim\mathcal{N}_{\mathbb{C}}(0, 1)$. The channel matrices of all K users are assumed to be perfectly known at the receiver, but not at the transmitter. At the receiver, sphere decoding [10] is used to conduct the maximum-likelihood decoding search.

III. New Multiuser STBCs

Let us consider all K users, assuming that a joint codeword matrix $\mathbf{X} \in \mathcal{C}$ is transmitted, it may occur that $\|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2 > \|\mathbf{Y} - \mathbf{H}\widehat{\mathbf{X}}\|^2$, with $\mathbf{X} \neq \widehat{\mathbf{X}}$, resulting in a *pairwise error*. Let $\mathbf{X} - \widehat{\mathbf{X}}$, with $\mathbf{X} \neq \widehat{\mathbf{X}}$, be the *joint codeword-difference matrix*, and let $\mathbf{A} \triangleq (\mathbf{X} - \widehat{\mathbf{X}})(\mathbf{X} - \widehat{\mathbf{X}})^\dagger$ be the *joint codeword-distance matrix*.

Similarly, when only user k is in error, assuming that a codeword matrix $\mathbf{C}_k \in \mathcal{C}_k$ is transmitted and $\widehat{\mathbf{C}}_k$ is erroneously detected at the receiver, we call $\mathbf{C}_k - \widehat{\mathbf{C}}_k$ the *user codeword difference matrix*. The corresponding *user codeword distance matrix* is defined as $\mathbf{E}^{(k)} \triangleq (\mathbf{C}_k - \widehat{\mathbf{C}}_k)(\mathbf{C}_k - \widehat{\mathbf{C}}_k)^\dagger$. Let r_k denote the minimum rank of $\mathbf{E}^{(k)}$ for all user codeword pairs in $\mathcal{C}_k$. We will assume $r_k = \min(n_t, N) = r$ for all k, i.e., all user codes have *full rank*.

If this full-rank condition holds for all K users, it is not guaranteed that the joint code is also full rank. We say that a multiuser STBC is *jointly full rank* if when all $\mathbf{E}^{(k)} \neq \mathbf{0}$, then $\mathrm{rank}(\mathbf{A}) = Kr$. Note that this property still holds for any subset of the K users. We will show in the following how to design such codes.

A. Jointly Full-Rank Design

Here, we assume that there are K users and $n_r = 2$ antennas at the receiver. Each user employs $n_t = 2$ transmit antennas. We choose $N = 2K$ channel uses so that the joint codeword matrix $\mathbf{X} \in \mathbb{C}^{n_t K \times N}$ is a square matrix.

Given the kth user information symbol vector $\mathbf{s}^{(k)}$, we use an algebraic unitary matrix $\mathbf{M}$ with *full diversity* [5], [7] to generate

$$\mathbf{v}^{(k)} = \mathbf{M}\mathbf{s}^{(k)} = \left[v_1^{(k)}, \dots, v_N^{(k)}\right]^T, \qquad k = 1, \dots, K \tag{3}$$

where the matrix $\mathbf{M}$ was originally given in [5] and [7]. The matrix $\mathbf{M}$ is obtained from the canonical embedding of an integral basis $\{\omega_j\}$, $j = 1, \dots, N$, of an ideal of an algebraic number field L of degree N over $\mathbb{Q}(i)$ [4]. Therefore, the matrix $\mathbf{M}$ guarantees the full-diversity property of $\mathbf{v}^{(k)}$, i.e., all the elements of $\mathbf{v}^{(k)}$ are nonzero for any nonzero information vector $\mathbf{s}^{(k)}$ [4].

Then, the user codewords of the K users for a 2 × 2 MIMO are generated in a cyclic order in (4), shown at the bottom of the next page, where γ a complex number on the unit circle (i.e., $|\gamma| = 1$) to preserve a uniform transmitted power from each antenna [8]. In such a manner, the code will not incur in a peak-to-average power penalty, since all entries are nonzero with the same average power (see the design example for details). The following two lemmas show how to further select the parameter γ in the aforementioned code.

Lemma 1: For $\gamma \neq 1$, the aforementioned user codes $\mathbf{C}_k$ are full rank, (i.e., $r = 2$) for all K users. ■

Proof: It is enough to show that the two rows of $\mathbf{C}_k$ are linearly independent, which is equivalent to saying they cannot be scalar multiples for any given nonzero information vector $\mathbf{s}^{(k)}$. This is the case due to the term $\gamma \neq 1$, which multiplies a different number of elements in each row. ■

Lemma 2: Let $N = n_t k$ so that the joint codeword matrices $\mathbf{x}$ are square. If γ is transcendental and $\gamma^t \neq 1$ for $t < n$, then the multiuser code $\mathcal{C}$ is jointly full rank. ■

Proof: In this proof, we assume that the readers are familiar with some basic algebraic number theory [4]. Looking at the structure of the $N \times N$ square codeword matrix $\mathbf{X}$, we note that the elements of the lower triangular part are multiplied by γ. It can easily be verified that the determinant of $\mathbf{X}$ is a polynomial $p_{\mathbf{X}}(\gamma)$ in the variable γ by using the well-known Laplace determinant expansion [9], i.e.,

$$p_{\mathbf{X}}(\gamma) = \det(\mathbf{X}) = \sum_{\pi} \mathrm{sign}(\pi) \prod_{i=1}^{N} x_{i,\pi(i)}$$

where the sum runs over all the permutations π, and sign (π) is the sign of the permutation [9]. Since the factor γ appears in the lower triangular part of the matrix, this polynomial has a degree $N-1$. The coefficient of the term γ^{N-1} is given by $x_{1,N} \cdot x_{2,1} \cdot x_{3,2} \cdots x_{N,N-1} \neq 0$. This is not zero due to the full-diversity rotation in (3), which yields vectors $\mathbf{v}^{(k)}$ with all nonzero entries. All the coefficients of $p_{\mathbf{X}}(\gamma)$ are generated from the full-diversity rotation matrix $\mathbf{M}$, and therefore, they are in the algebraic number field L, as defined after (3).

It is well known that the roots of the polynomial equation $p_{\mathbf{X}}(\gamma) = 0$ are in some algebraic extension L' of L of degree at most $N-1$ [4]. For example, the polynomial equation $x^2 - 2 = 0$ has coefficients in the field $L = \mathbb{Q}$ and roots $\pm\sqrt{2} \in L' = \mathbb{Q}(\sqrt{2})$, which is an algebraic extension of degree 2 of $\mathbb{Q}$. However, to obtain a full-diversity code, we must have $p_{\mathbf{X}}(\gamma) = \det(\mathbf{X}) \neq \mathbf{0}$ for any $\mathbf{X}$, which can be guaranteed by choosing γ not in the algebraic extension L'. This is simply obtained by using a transcendental number γ, which, by definition, does not belong to any finite algebraic extension. ■

Note that Lemma 2 gives only a necessary condition, and some specific algebraic number not belonging to L' could also yield a jointly full-rank multiuser code.

B. Design Criteria

To simplify analysis, we assume that the jointly full-rank multiuser STBC is *linear* [6]. Then, the error probability of the multiuser MIMO is upper bounded by the following UB [3]:

$$P(e) \leq \sum_{\mathbf{X}\neq\mathbf{0}} \sum_{k=1}^{K} \sum_{(i_1,\ldots,i_k)}^{A_k} P(e_{i_1} \cap \cdots \cap e_{i_k} | \mathbf{X})$$

where e_k represents the kth user error event, and the sum $\sum_{(i_1,\ldots,i_k)}^{A_k}$ is over all $A_k \triangleq \binom{K}{k}$ possible k-tuples of users in error. The k-tuple $(i_1,\ldots,i_k)$ denotes the indices of k distinct users in error. Using the Chernoff bound, we can upper bound each term in the aforementioned UB $P(e)$ with

$$P(e_{i_1} \cap \cdots \cap e_{i_k} | \mathbf{X}) \leq \left(\frac{E_s}{N_0}\right)^{-n_r k r} \left[\delta_{(i_1,\ldots,i_k)}(\mathbf{X})\right]^{-n_r} \tag{5}$$

where $E_s \triangleq (1/K n_t N) \sum_{i,j} \mathbb{E}[|x_{i,j}|^2]$ is the average energy per QAM information symbol, and

$$\delta_{(i_1,\ldots,i_k)}(\mathbf{X}) \triangleq \det\left(\sum_{\ell=1}^{k} \mathbf{C}_{i_\ell} \mathbf{C}_{i_\ell}^{\dagger}\right) \tag{6}$$

are the joint determinants for the k-tuples $(i_1,\ldots,i_k)$. Let $\delta_k^{(\min)}$ be the corresponding *minimum determinants* $\delta_{(i_1,\ldots,i_k)}(\mathbf{X})$ among all the k-tuples. Finally, we consider a truncated UB based only on the terms corresponding to the minimum determinants $\delta_k^{(\min)}$, i.e.,

$$P(e) \approx \sum_{k=1}^{K} A_k B_k P\left(\delta_k^{(\min)}\right)$$

where the $A_k B_k$ is the total multiplicity of the term

$$P\left(\delta_k^{(\min)}\right) = \left(\frac{E_s}{N_0}\right)^{-n_r k r} \left(\delta_k^{(\min)}\right)^{-n_r} \tag{7}$$

which represents the dominant error probability of a k-tuple of users. In particular, B_k denotes the associate multiplicity of (7) for a given A_k.

The codes designed in the previous section satisfy the following lemma.

Lemma 3: Given the code (4), the determinants in (6) are all nonzero. ■

$$\begin{aligned}
\mathbf{C}_1 &= \begin{bmatrix} v_1^{(1)} & v_2^{(1)} & \cdots & & v_N^{(1)} \\ \gamma v_N^{(1)} & v_1^{(1)} & & \cdots & v_{N-1}^{(1)} \end{bmatrix} \\
\mathbf{C}_2 &= \begin{bmatrix} \gamma v_{N-1}^{(2)} & \gamma v_N^{(2)} & \cdots & & v_{N-2}^{(2)} \\ \gamma v_{N-2}^{(2)} & \gamma v_{N-1}^{(2)} & \gamma v_N^{(2)} & \cdots & v_{N-3}^{(k)} \end{bmatrix} \\
&\vdots \qquad\qquad \ddots \quad \ddots \\
\mathbf{C}_k &= \begin{bmatrix} \gamma v_{N-(n_t k - n_t)+1}^{(k)} & \cdots & \gamma v_N^{(k)} & & \cdots & v_{N-(n_t k - n_t)}^{(k)} \\ \gamma v_{N-(n_t k - n_t)}^{(k)} & \gamma v_{N-(n_t k - n_t)+1}^{(k)} & \cdots & \gamma v_N^{(k)} & \cdots & v_{N-(n_t k - n_t + 1)}^{(k)} \end{bmatrix} \\
&\vdots \qquad\qquad \ddots \quad \ddots \\
\mathbf{C}_K &= \begin{bmatrix} \gamma v_{N-(n_t K - n_t)+1}^{(K)} & \cdots & \gamma v_N^{(K)} & & \cdots & v_{N-(n_t K - n_t)}^{(K)} \\ \gamma v_{N-(n_t K - n_t)}^{(K)} & \gamma v_{N-(n_t K - n_t)+1}^{(K)} & \cdots & \gamma v_N^{(K)} & \cdots & v_{N-(n_t K - n_t + 1)}^{(K)} \end{bmatrix}
\end{aligned} \tag{4}$$

TABLE I
COMPARISON OF MINIMUM DETERMINANTS WHEN ONE OR TWO USERS ARE IN ERROR, ASSOCIATED MULTIPLICITIES, AND 4-QAM SIGNALING

Codes	$\delta_1^{(\min)}$	A_1B_1	$\delta_2^{(\min)}$	A_2B_2
New	60.93	32	61.96	64
GB	16	64	32	256
ZL	4	64	8	256

Proof: Since the terms $\mathbf{C}_{i_\ell}\mathbf{C}_{i_\ell}^\dagger$ in (6) are positive definite, we use the determinant inequality

$$\det\left(\sum_{\ell=1}^{k}\mathbf{C}_{i_\ell}\mathbf{C}_{i_\ell}^\dagger\right) \geq \sum_{\ell=1}^{k}\det\left(\mathbf{C}_{i_\ell}\mathbf{C}_{i_\ell}^\dagger\right) > 0$$

where the determinants on the right-hand side are all greater than zero due to Lemma 1. ■

Hence, under the full-rank and linearity assumption, to minimize the error probability $P(e)$, we should design multiuser STBCs to

1) maximize the minimum determinants $\delta_k^{(\min)} \backslash \forall k$;
2) minimize the associated multiplicity A_kB_k.

C. Design Example

As an example, we consider $K = 2$ users, each of which employs a 2×2 MIMO over a quasi-static fading channel. The codeword matrix of each user spans $N = 4$ channel uses using the following matrix:

$$\mathbf{M} = \begin{bmatrix} 0.26-0.31i & 0.35-0.42i & -0.42+0.51i & -0.21+0.26i \\ 0.26+0.09i & 0.47+0.16i & 0.16+0.05i & 0.76+0.26i \\ 0.26+0.21i & -0.51-0.42i & -0.42-0.36i & 0.31+0.26i \\ 0.26-0.76i & -0.05+0.16i & 0.16-0.47i & -0.09+0.26i \end{bmatrix}. \tag{8}$$

We choose the transcendental number $\gamma = \exp(i\lambda)$ in Lemma 2, where $\lambda \in [0, \pi]$ is any rational number (the proof that γ is indeed transcendental can be found in [8].

We conduct a search over the values of λ in steps of 0.1 to maximize the minimum determinant $\delta_k^{(\min)}$ $\forall k$ and minimize the associated multiplicity A_kB_k, with 4-QAM signaling. The choice of the step is related to the sensitivity of the performance with respect to λ and is based on experimental verification. We found that the best choice of λ is 3 for 4-QAM signaling, and subsequently, we use this value for 16-QAM signaling. The resulting code is compared with previously known codes in [1] and [3] in Table I for 4-QAM signaling. We note that the proposed code and the known codes in [1] and [3] are "full-rank" joint multiuser STBCs, i.e., $r_k = 2$ and rank $(\mathbf{A}) = 4$.

We see that when one user is in error (condition 1) and when both users are in error (condition 2), the minimum determinants of our code are the largest. In both conditions, the associated multiplicities of the proposed code are significantly smaller than those of [1] and [3].

We compare the performance of the proposed code and the known codes of [1] and [3] in Figs. 1 and 2, for 4-QAM and 16-QAM signaling, respectively. In both figures, the error probability $P(e)$ takes into account the total number of errors of both users. The *peak-signal-to-noise ratio* is defined as Peak$-$SNR $\triangleq n_tE_p/N_0$, where $E_p = \max_{i,j}\mathbb{E}[|x_{i,j}|^2]$ denotes the *peak average energy* of a transmitted QAM symbol from one antenna. We have $E_p = E_s$ for the proposed code and the one in [1], whereas $E_p = 2E_s$ for the code in [3], which has some zero entries in the codewords.

In Fig. 1, we can see that at $P(e) = 10^{-3}$, the proposed code outperforms the codes of [1] and [3] by 0.2 and 3.1 dB, respectively.

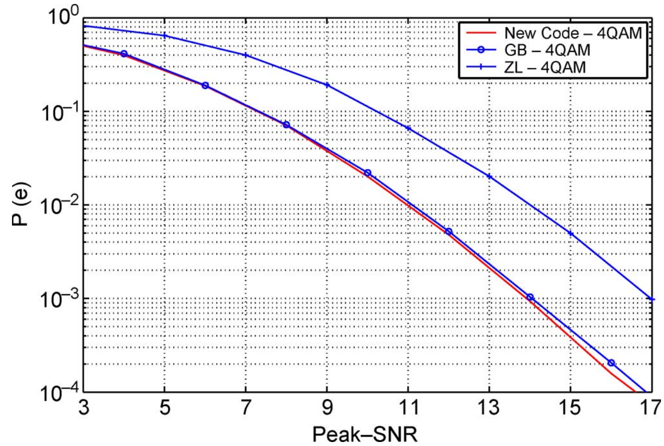

Fig. 1. Comparison of the $P(e)$ performance of the new code, known codes in [1] and [3] (i.e., GB and ZL, respectively), 4-QAM signaling, and quasi-static fading channel.

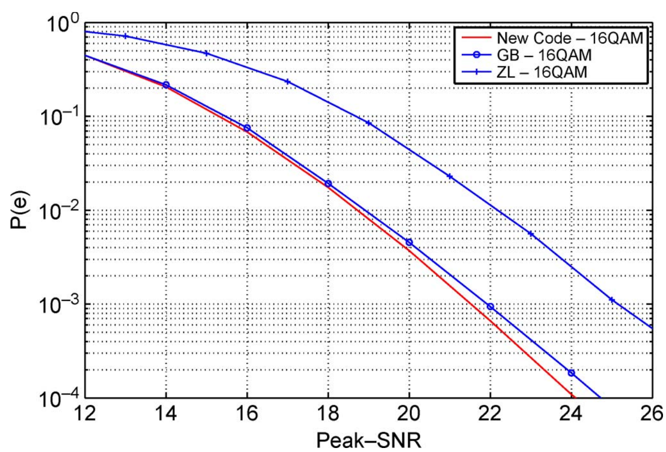

Fig. 2. Comparison of the $P(e)$ performance of the new code, known codes in [1] and [3], 16-QAM signaling, and quasi-static fading channel.

In Fig. 2, we observe that the proposed code outperforms the codes of [1] and [3] by 0.6 and 3.6 dB, respectively.

IV. CONCLUSION

In this paper, we have proposed new algebraic multiuser 2×2 STBCs for quasi-static MIMO MACs based on full-diversity algebraic rotations. For any number of users, we have shown how to design a family of jointly full-rank STBCs to minimize the truncated UB. It has been shown that the proposed multiuser STBCs do not incur in any peak-to-average penalty while outperforming all previously known codes.

ACKNOWLEDGMENT

The authors would like to thank M. E. Gärtner and H. Bolcskei for their fruitful discussions and the anonymous reviewers for the constructive comments.

REFERENCES

[1] M. E. Gärtner and H. Bölcskei, "Multiuser space–time/frequency code design," in *Proc. IEEE ISIT*, Seattle, WA, Jul. 2006, pp. 2819–2823.
[2] M. Badr and J.-C. Belfiore, "Optimal space–time codes for the noncooperative MAC channel," in *Proc. Int. Zurich Semin.*, Zurich, Switzerland, Mar. 2008, pp. 132–135.

[3] W. Zhang and K. B. Letaief, "A systematic design of multiuser space–frequency codes for MIMO-OFDM systems," in *Proc. IEEE ICC*, Jul. 2007, pp. 1054–1058.

[4] F. Oggier and E. Viterbo, "Algebraic number theory and code design for Rayleigh fading channels," *Commun. Inform. Theory*, vol. 1, no. 3, pp. 333–415, Dec. 2004.

[5] F. Oggier, G. Rekaya, J.-C. Belfiore, and E. Viterbo, "Perfect space–time block codes," *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3885–3902, Sep. 2006.

[6] E. Biglieri, Y. Hong, and E. Viterbo, "On fast-decodable space–time block codes," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 516–522, Feb. 2009.

[7] J. Boutros and E. Viterbo, "Signal space diversity: A power- and bandwidth-efficient diversity technique for the Rayleigh fading channel," *IEEE Trans. Inf. Theory*, vol. 44, no. 4, pp. 1453–1467, Jul. 1998.

[8] M. O. Damen, A. Tewfik, and J.-C. Belfiore, "A construction of a space–time code based on number theory," *IEEE Trans. Inf. Theory*, vol. 48, no. 3, pp. 1–12, Mar. 2002.

[9] *Wikipedia*. [Online]. Available: http://en.wikipedia.org/wiki/Expansion_by_minors

[10] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1639–1642, Jul. 1999.

Multiuser Detection for Asynchronous Broadband Single-Carrier Transmission Systems

Koichi Ishihara, *Member, IEEE*, Yasushi Takatori, *Member, IEEE*, Shuji Kubota, *Member, IEEE*, and Fumiyuki Adachi, *Fellow, IEEE*

***Abstract*—The multiuser multiple-input–multiple-output (MU-MIMO) technique is an attractive candidate in improving spectrum efficiency. However, in the uplink, the difference in the transmit timing among the signals from multiple mobile terminals (MTs) becomes significant. If the arrival timing difference among MTs exceeds the cyclic prefix (CP) length, it is difficult to separate the signal of each MT, and the transmission quality is severely degraded due to the effect of interblock interference (IBI). In this paper, we propose a novel multiuser detection (MUD) technique that can separate the signals, regardless of the arrival timing differences between the MTs. The proposed MUD technique is performed in the frequency domain by overlapping the fast Fourier transform (FFT) blocks, and the technique can suppress the effect of the IBI while retaining a low complexity level. The achievable bit error rate (BER) performance is evaluated by computer simulation in a frequency-selective fading channel, and the results confirm the effectiveness of the proposed technique.**

***Index Terms*—Cyclic prefix (CP), multiuser detection (MUD), multiuser multiple-input multiple-output (MU-MIMO).**

Manuscript received March 3, 2008; revised August 18, 2008 and December 1, 2008. First published January 13, 2009; current version published May 29, 2009. This work was supported in part by the Ministry of Internal Affairs and Communications, Japan, under "Research and Development of Fundamental Technologies for Advanced Radio Frequency Spectrum Sharing in Mobile Communication Systems" Grant. The review of this paper was coordinated by Dr. C. Cozzo.

K. Ishihara and Y. Takatori are with the NTT Network Innovation Laboratories, Nippon Telegraph and Telephone Corporation, Yokosuka 239-0847, Japan (e-mail: ishihara.koichi@lab.ntt.co.jp; takatori.yasushi@lab.ntt.co.jp).

S. Kubota is with the Department of Electrical Communications, College of Engineering, Shibaura Institute of Technology, Tokyo 135-8548, Japan (e-mail: kubota.shuji@lab.ntt.co.jp).

F. Adachi is with the Department of Electrical and Communications Engineering, Tohoku University, Sendai 980-8579, Japan (e-mail: adachi@ecei.tohoku.ac.jp).

Digital Object Identifier 10.1109/TVT.2009.2012716

I. Introduction

Since many of the next-generation wireless access systems are expected to provide broadband services while using limited radio frequency resources, spectrum efficiency must be improved. The multiuser multiple-input–multiple-output (MU-MIMO) technique [1] is an attractive candidate in addressing this problem. In MU-MIMO, multiple mobile terminals (MTs) generate a virtual large antenna array, and therefore, a large MIMO effect is expected, even for simple MTs. Many studies on MU-MIMO systems in both the uplink [2], [3] and downlink [4], [5] have suggested that a higher channel capacity can be achieved. In this paper, we focus on uplink MU-MIMO systems. Orthogonal frequency-division multiplexing (OFDM) [6], [7] and single carrier with frequency-domain equalization (SCFDE) [8]–[10] techniques are commonly applied to broadband wireless access systems in MU-MIMO systems. OFDM and SCFDE systems employ a cyclic prefix (CP) to induce the effect of a frequency-selective channel into the circular convolution of a transmitted signal and the channel impulse responses. SCFDE, in particular, attains a transmission quality level that is comparable with that of OFDM while achieving a lower peak-to-average power ratio (PAPR) [10]. Since a simple MT is expected to use a low-cost amplifier, particularly in uplink MU-MIMO systems, the low PAPR characteristic of a single carrier (SC) allows the MT to use a high transmit power without signal distortion.

Basically, we can regard an uplink MU-MIMO system as a single-user MIMO (SU-MIMO) system when beamforming is not carried out at the transmitters. However, there are some differences between MU- and SU-MIMO systems. In MU-MIMO systems, although it is impossible to perfectly coordinate among multiple MTs in actual wireless access systems, the access point (AP) must cope with both frequency and timing offsets. To overcome the frequency-offset problem, we proposed a space–time equalization method for MU-MIMO systems [11]. In this paper, we focus on the timing-offset problem. In the uplink, the arrival timing differences at an AP among the signal streams from multiple MTs caused by the differences in the geographical distance between the AP and each MT represent a significant problem. When the transmit timing control is used and all the arrival timing differences are within the CP period, there is no interblock interference (IBI), and the transmitted signal can be recovered. However, if the arrival timing differences exceed the CP length due to imperfect timing control, the transmission quality degrades. Furthermore, when the transmit timing control is not used and all the MTs asynchronously transmit their signals, the arrival timing differences are uniformly distributed and exceed the CP period. This significantly degrades the transmission quality. Although various cancellation techniques were proposed [12], [13] to suppress the IBI caused by an insufficient CP length, these methods require a decision feedback loop, resulting in a high computational complexity level. It is clear that time-domain equalization techniques such as that described in [14] can also be applied. However, since the number of taps in the time-domain equalizer increases in broadband wireless access systems, the complexity of the equalizer becomes prohibitively high.

In this paper, we propose a novel multiuser detection (MUD) method that can be used for asynchronous MU-MIMO systems in a frequency-selective fading channel. The proposed MUD technique is performed in the frequency domain with overlapping of the fast Fourier transform (FFT) blocks while retaining a low complexity level. The proposed method does not require transmit timing control and a CP. SC transmission employing overlap frequency-domain equalization (FDE) was reported in [15]–[17], which requires no CP and provides almost the same bit error rate (BER) performance as that for SCFDE