Information Theoretic Analysis of Bit-Interleaved Time Space Codes for Fading Channels

Ezio Biglieri Giorgio Taricco Emanuele Viterbo

Dipartimento di Elettronica
Politecnico di Torino
Torino, Italy

International Symposium on Information Theory and Its Applications

Honolulu, Hawaii, November 5–8, 2000

Introduction

- We study a multiple antenna system with t TX and r RX antennas in fast and slow fading
- Each receiver sees the signals radiated from the t transmit antennas
- To assess the system quality we consider channel capacity or ε-capacity
- These are a limit to the error free bit rate
- We compare a practical scheme using BIMC and Turbo Codes to these limits

System model

$$\mathbf{y}[\ell] = \mathbf{H}[\ell]\mathbf{x}[\ell] + \mathbf{n}[\ell], \qquad \ell = 1, \dots, L$$

- $\mathbf{y}[\ell]$ signal received at discrete time ℓ ($r \times 1$ vector)
- $\mathbf{n}[\ell]$ white noise ($r \times 1$ vector)
- $\mathbf{x}[\ell]$ signals transmitted at time ℓ from constellation $\mathcal{X}$ ($t \times 1$ vector)
- $$\mathbf{H}[\ell] = \begin{bmatrix} \mathbf{h}_1[\ell] \\ \mathbf{h}_2[\ell] \\ \vdots \\ \mathbf{h}_r[\ell] \end{bmatrix} \qquad r \times t \text{ channel-gain matrix}$$
- $\mathcal{X}_i$ constellations whose signals are all possible values of $\mathbf{h}_i[\ell]\mathbf{x}[\ell]$.

Fading channel models

- $\mathbf{H}[\ell] = \mathbf{H}$ is random, but remains constant during the transmission of a code word ("block-fading" or "slow-fading" channel).
- $\mathbf{H}[\ell]$ takes on independent values for each ℓ ("independent-fading" or "fast-fading" channel).
- In both cases h_{ij} independent, circularly complex Gaussian distributed with zero mean and variance $\mathbb{E}[|h_{ij}|^2] = 1$
  1. Rayleigh fading
  2. Transmit and the receive antennas have sufficient spatial separation

Capacity (fast-fading)

- With fast fading, we may assume that the channel is ergodic, and hence its capacity is defined as

$$C = \sup_{p(\mathbf{x})} \mathbb{E}[\mathcal{I}(\mathbf{x};\mathbf{y} \mid \mathbf{H})], \quad \mathbb{E}[\mathbf{x}^\dagger \mathbf{x}] \leq \mathcal{P} \tag{1}$$

where $\mathcal{I}(\mathbf{x};\mathbf{y} \mid \mathbf{H})$ is the *conditional mutual information* between input and output, $p(\mathbf{x})$ is the probability density of the input, and $\mathcal{P}$ is the maximum power across all transmit antennas.

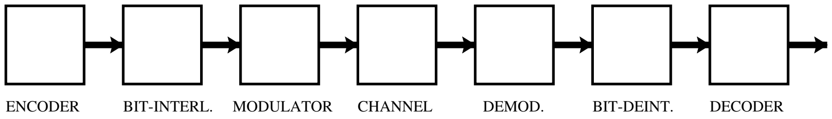
Outage Probability and ε-capacity (slow-fading)

- For slow-fading, given the transmission rate R we define the *outage probability* as

$$P_{\text{out}}(R) = \mathbb{P}[\sup_{p(\mathbf{x})} \mathcal{I}(\mathbf{x};\mathbf{y}|\mathbf{H}) < R] \tag{2}$$

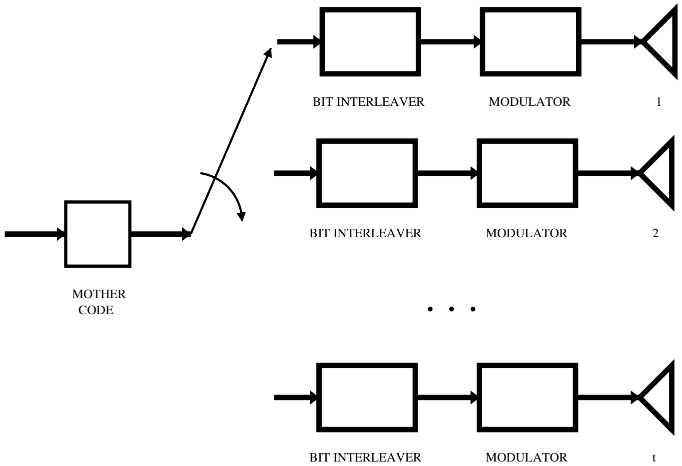
- This represents the probability that the maximum conditional mutual information falls below R due to channel variations.
- In the presence of an outage, transmission is totally unreliable irrespective of the code used.
- We define the *ε-capacity* of the channel, as the maximum achievable rate $R = R^*(\varepsilon)$ for which $P_{\text{out}}(R) < \varepsilon$.

BICM



- Interleaving is done at the bit level
- Demodulation (bit metric computation) is separated from decoding
- BICM transforms a channel into parallel binary channels with a minor degradation with respect to classical Coded Modulation (CM)
- Hamming distance vs. Euclidean distance

BICM with Multiple Antennas



Capacity Calculations

- Telatar studied the Gaussian input case. We are interested in discrete input signal sets $\mathcal{X}$, with uniform distribution.
- Transition probabilities $p(\mathbf{y} \mid \mathbf{x}, \mathbf{H}) = (\pi N_0)^{-t} \exp(-\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 / N_0)$
- Coded modulation (CM)

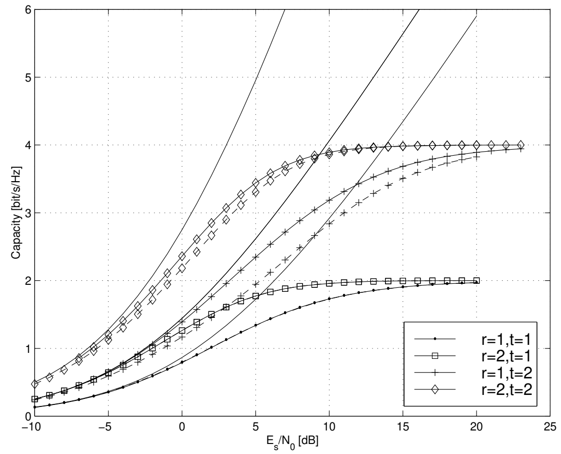
$$C = mt - \mathbb{E}_{\mathbf{x},\mathbf{y},\mathbf{H}} \left[\log_2 \frac{\sum_{\mathbf{z} \in \mathcal{X}^t} p(\mathbf{y}|\mathbf{z},\mathbf{H})}{p(\mathbf{y}|\mathbf{x},\mathbf{H})} \right]$$

- Bit-iterleaved Coded Modulation (BICM)

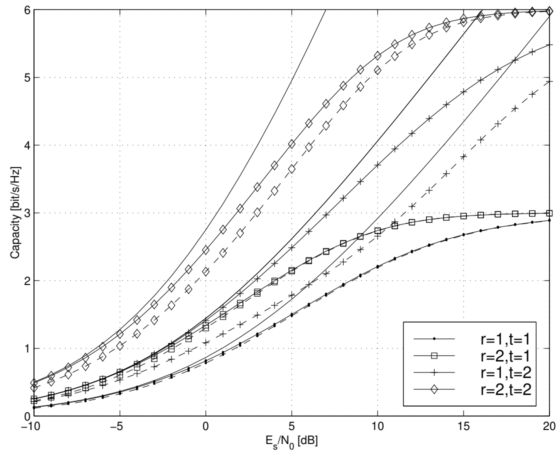
$$C = mt - \sum_{i=1}^{mt} \mathbb{E}_{\mathbf{x},\mathbf{y},\mathbf{H},b} \left[\log_2 \frac{\sum_{\mathbf{z} \in \mathcal{X}^t} p(\mathbf{y}|\mathbf{z},\mathbf{H})}{\sum_{\mathbf{z} \in \mathcal{X}^t_{i,b}} p(\mathbf{y}|\mathbf{z},\mathbf{H})} \right]$$

where $\mathcal{X}^t$ denotes the t-fold Cartesian product of $\mathcal{X}$, and $\mathcal{X}^t_{i,b}$ denotes the subset of signals from $\mathcal{X}^t$ whose ith bit is equal to b.

Capacity with QPSK



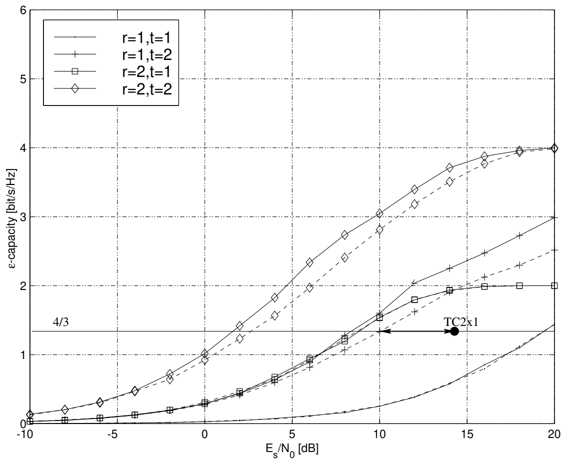
Capacity with 8-PSK



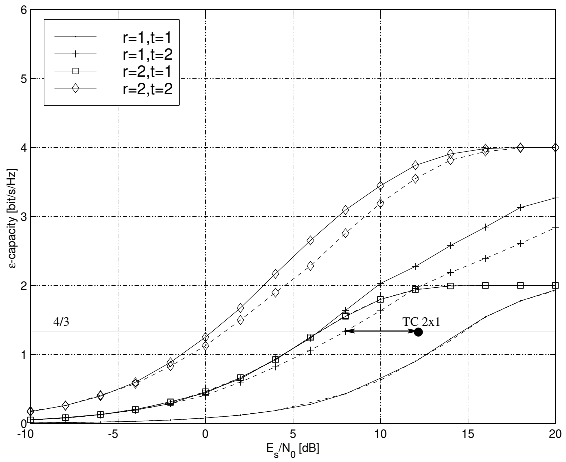
Evaluation of a practical scheme

- Consider the slow fading case.
- We compare a specific coding scheme with finite length codewords with the ultimate limits given by the ε-capacity curves.
- We evaluate the distance on the ε-capacity curves in terms of E_s/N_0 dB by looking at the simulated Frame Error Rate (FER) curves of the practical coding scheme.
- In this way, it is possible to evaluate the goodness of any coding scheme CM or BICM.

ε-capacity with QPSK ($\varepsilon = 0.02$)



ε-capacity with QPSK ($\varepsilon = 0.05$)



Example

BICM using $R = 1/3$ turbo coded QPSK

- frames with 416 information bits
- spectral efficiency $\eta = 4/3$ bit/s/Hz

To make a fair comparison in the two cases:

- $L = 312$ channel uses at rate R_s for $(t = 2, r = 1)$
- $L = 624$ channel uses at rate $2R_s$ for $(t = 1, r = 1)$ (double bandwidth)

Comparison with the ε-capacity curves we find the $E_s/N_0 = \eta E_b/N_0$ values giving a FER=ε. For $t = 2, r = 1$ we have FER=0.02 at $E_s/N_0 = 14.25$ dB and FER=0.05 at $E_s/N_0 = 12.25$ dB.

Turbo codes rate 1/3