Signal-Space Coding for the Collision Channel: Collision Resistant Modulation

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Presentation outline

- Introduction
- The multiple access slotted collision channel
- Collision Resistant Modulation
- Error Probability Analysis
- Results
- Conclusions and future work

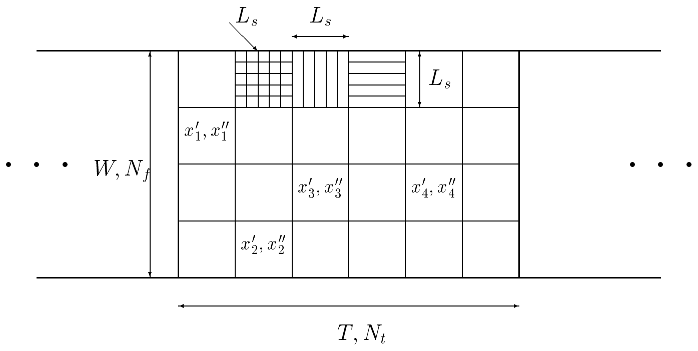
Introduction

- Multi-user, random access, time-frequency hopping, wireless network.
- Collisions occur among different users accessing the same channel slots, (e.g. as in a slotted ALOHA protocol).
- Signal-space code with D-dimensional signals. Each component is sent over a different channel slot and can be independently affected by a collision.
- CRMs have the property that even if $k < D$ slots collide the signal can still be correctly detected, in the absence of noise, from its $D - k$ un-collided components.

The multiple access slotted collision channel

- N_u number of active users
- $N_s = N_t \times N_f$ number of available slots where N_f is the number of carriers and N_t is the number of time slots per frame
- L_s number of real dimensions per slot
- $N_s L_s = 2WT$ where T is the time duration of a frame and W is the overall system bandwidth
- R_b [bits/frame] is the user transmission bit rate
- η [bits/dim] is the spectral efficiency of the modulation scheme
- $g = R_b/(\eta L_s)$ is the number slots per frame occupied by a user

The frame structure



$L_s = 2 \quad D = 4 \quad \mathbf{x}' = (x_1', \ldots x_4')^T \quad \mathbf{x}'' = (x_1'', \ldots x_4'')^T$

The multiple access slotted collision channel

- We model the transmission process of each user in each slot, as a Bernoulli process with parameter $p = g/N_s = R_b/(2WT\eta)$ representing the probability that one user occupies one slot.
- The probability of a collision in one slot is
$$P_{col} = 1 - (1-p)^{N_u - 1}$$
- In the limit case for $N_s, N_u \to \infty$ (i.e. infinite users population) the number of transmitting users in each slot becomes a Poisson distributed random variable with average $G = \lim_{N_s, N_u \to \infty} gN_u/N_s$ and
$$P_{col} = 1 - e^{-G}$$
- The parameter G corresponds to the so called *offered channel traffic*.

The Collision+Fading Vector Channel Model

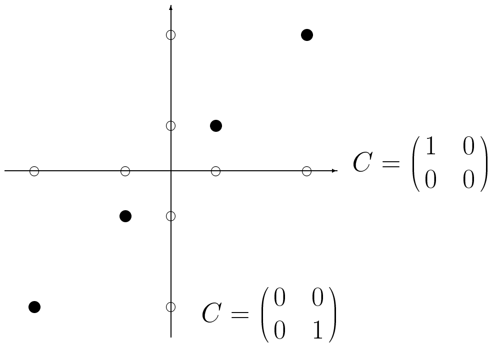
$$\boxed{\mathbf{y} = \mathrm{C}(\mathrm{G}\mathbf{x} + \mathbf{n})}$$

- $\mathbf{y} = (y_1, \ldots y_D)^T$ is a received vector
- $\mathbf{x} = (x_1, \ldots x_D)^T$ is a transmitted vector taken from the signal set (signal-space code) S of cardinality $|S|$
- $\mathbf{n} = (n_1, \ldots n_D)^T$ is additive white Gaussian noise with i.i.d. components $n_i = \mathcal{N}(0, N_0/2)$
- $G = diag(g_1, \ldots g_D)$ represents the channel fading coefficients
- $C = diag(c_1, \ldots c_D)$ represents the collision pattern and $c_i = 0$ or 1 are Bernoulli i.i.d. random variables with $P_{col} = P(c_i = 0)$.

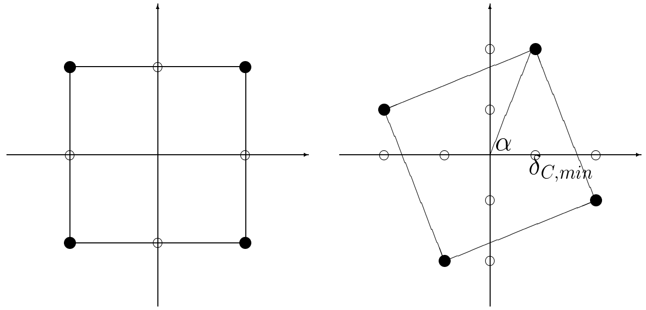
Collision Resistant Modulation

- The receiver has perfect channel state information (CSI) on both the collisions and the fading. ML detection is obtained for
$$\min_{\mathbf{x} \in S} d_C^2(\mathbf{x}, \mathbf{y}) = \min_{\mathbf{x} \in S} \sum_{i=1}^{D} c_i (y_i - g_i x_i)^2$$
- Let $S(C)$ be the projection of S on the subspace generated by the $D - k$ axes corresponding to the non zero c_i's
- A **Collision Resistant Modulation** (CRM) is a signal-space code S with the property that any projection on any coordinate subspace has the same number of distinct points as the original set, i.e., $|S(C)| = |S|$ for all non-zero collision patterns C.

Collision Resistant Modulation – Example 1



Collision Resistant Modulation – Example 2



Error Probability Analysis

- Union Bound on the *symbol error probability*

$$P(e) \leq \frac{1}{|S|} \sum_{\mathbf{x} \in S} \sum_{\mathbf{x} \neq \hat{\mathbf{x}}} E_C[P(\mathbf{x} \to \hat{\mathbf{x}}|C)]$$

  where $P(\mathbf{x} \to \hat{\mathbf{x}}|C)$ is the pairwise error probability averaged with respect to G and conditioned by C, and E_C denotes the expectation over all the possible 2^D collision patterns.

- Note that $P(C) = P_{col}^k (1 - P_{col})^{D-k}$ where k is # collisions in C.
- For the Gaussian channel, G is the identity matrix and we get

$$P(\mathbf{x} \to \hat{\mathbf{x}}|C) = \frac{1}{2} \mathrm{erfc} \left(\frac{d_C(\mathbf{x}, \hat{\mathbf{x}})}{2\sqrt{N_0}} \right)$$

- Good CRMs for the Gaussian channel should maximize the minimum distance $d_{C,min}$ in the projected constellations $S(C)$.

Error Probability with Rayleigh Fading

- In the case of an independent real Rayleigh fading channel, the g_i's are real i.i.d. Rayleigh random variables with unit second moment

$$P(\mathbf{x} \to \hat{\mathbf{x}}|C) = \frac{1}{2} \sum_{i=1}^{D-k} \left(1 - \frac{\delta_{C,i}}{\sqrt{4N_0 + \delta_{C,i}^2}} \right) \prod_{\substack{j=1 \\ j \neq i}}^{D-k} \frac{\delta_{C,i}^2}{\delta_{C,i}^2 - \delta_{C,j}^2} \leq \frac{(4N_0)^{D-k}}{\prod_{i=1}^{D-k} \delta_{C,i}^2}$$

  where $\delta_{C,i} = |x_i^{(C)} - \hat{x}_i^{(C)}|$, for $i = 1, \ldots D-k$, are the component-wise distances between the projections $\mathbf{x}^{(C)}$ and $\hat{\mathbf{x}}^{(C)}$ and k the number of collisions in C

- A good CRM for the fading channel would also be reasonably good for the Gaussian channel but not vice versa, since

$$d_C^2(\mathbf{x}, \hat{\mathbf{x}}) = \sum_{i=1}^{D-k} \delta_{C,i}^2$$

Results and comparisons for Gaussian and Fading channels

- Let the offered channel traffic $G = gN_u/N_s = H/\eta$, then $H = N_u R_b/(L_s N_s)$ is the overall system spectral efficiency ([bit/s/Hz]) and it depends only on the system parameters and not on the type of signal set used for transmission.
- We compare four schemes having the same H and $R_b = 4$[bit/frame].
  1. CRM-1: 4D diagonal signal set, $\eta = 1$[bit/dim], $g = 4$, $D = 4$
  2. CRM-2: rotated 4D hyper-cube, $\eta = 1$[bit/dim], $g = 4$, $D = 4$
  3. 2-PAM: $\eta = 1$ [bit/dim], $g = 4$, $D = 1$
  4. 16-PAM: $\eta = 4$ [bit/dim], $g = 1$, $D = 1$
- $L_s = 1$, so that we compare the 4D symbol error probabilities.

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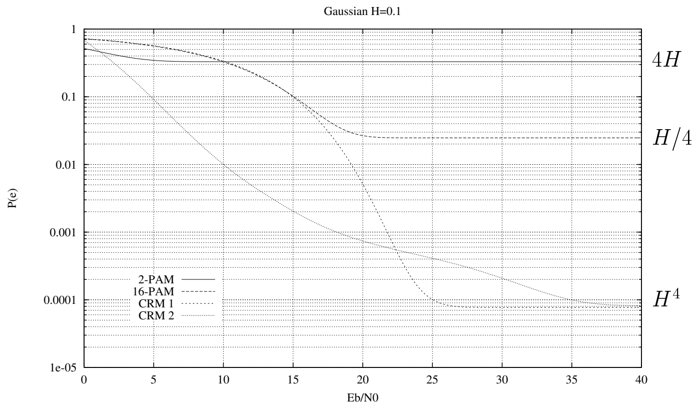
Symbol error probability – Gaussian $H = 0.1$



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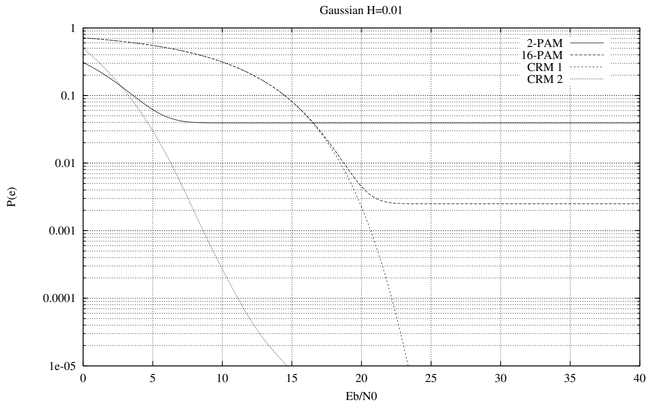
Symbol error probability – Gaussian $H = 0.01$



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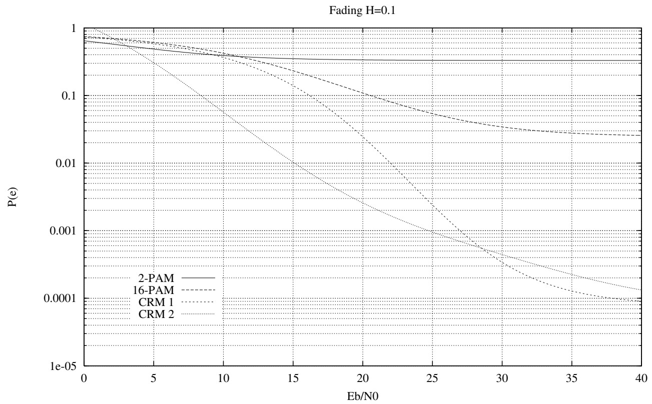
Symbol error probability – Fading $H = 0.1$



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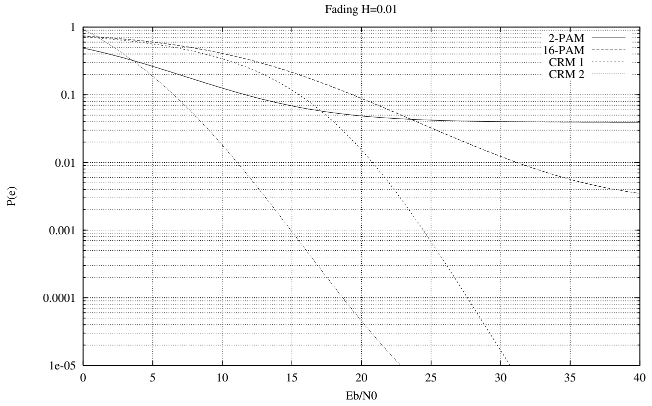
Symbol error probability – Fading $H = 0.01$



Conclusions and Future Work

- The scheme we propose may be applied for control packet transmission in a wireless network, a case where random access is essential and the channel is lightly loaded.
- New CRM designs with algebraic constructions
- Outer coding schemes to improve the packet error rates
- Improved protocols which take further advantages of CRM