POLYNOMIAL CANCELLATION CODING OF OFDM TO REDUCE INTERCARRIER INTERFERENCE DUE TO DOPPLER SPREAD

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ABSTRACT

Polynomial cancellation coding (PCC) is a coding method for orthogonal frequency division multiplexing (OFDM) in which the information to be transmitted is modulated onto weighted groups of subcarriers rather than onto individual subcarriers. It has previously been shown that PCC substantially reduces the sensitivity of OFDM to carrier frequency offset. In this paper, it is shown that PCC also reduces the intercarrier interference (ICI) due to Doppler spread. Results are presented for a frequency non-selective i.e. flat fading channel, subject to classical Doppler spread. By using weighted pairs of subcarriers the ICI due to Doppler spread can be reduced by approximately 15dB. By using weighted groups of three subcarriers a further 15dB reduction in ICI can be achieved.

INTRODUCTION

OFDM is a popular technique for broadcast channels where the single wideband transmission is replaced by many parallel narrowband transmissions [1,2]. In an earlier paper [3,4,5] the ICI caused by carrier frequency offset in OFDM was analysed. It was shown that this ICI could be very much reduced by using PCC, a technique in which weighted groups of subcarriers, rather than individual subcarriers, are modulated. These results are summarized here.

POLYNOMIAL CANCELLATION CODING

Figure 1 shows the block diagram of an OFDM system. The diagram is general enough to describe both normal OFDM, and OFDM with PCC coding. The high-speed data to be transmitted is divided into n lower speed parallel channels. The data in the k-th parallel channel in the i-th symbol period is represented by $d_{k,i}$. This will in general be a complex value. The data values $d_{0,i} \cdots d_{n-1,i}$ determine the values $a_{0,i} \cdots a_{N-1,i}$ which modulate the N subcarriers in the i-th symbol period. For normal OFDM $n = N$, and $a_{k,i} = d_{k,i}$; one data value is used to modulate each subcarrier. With PCC, the data to be transmitted is mapped onto weighted groups of subcarriers. For example, to apply PCC to pairs of subcarriers, the subcarriers in each pair must have relative weightings +1 and –1. In this case $n = N/2$. The first data value in each symbol period is used to modulate the first two subcarriers: $a_{0,i} = d_{0,i}$, $a_{1,i} = -d_{0,i}$. The second data value modulates the third and fourth subcarrier and so on.

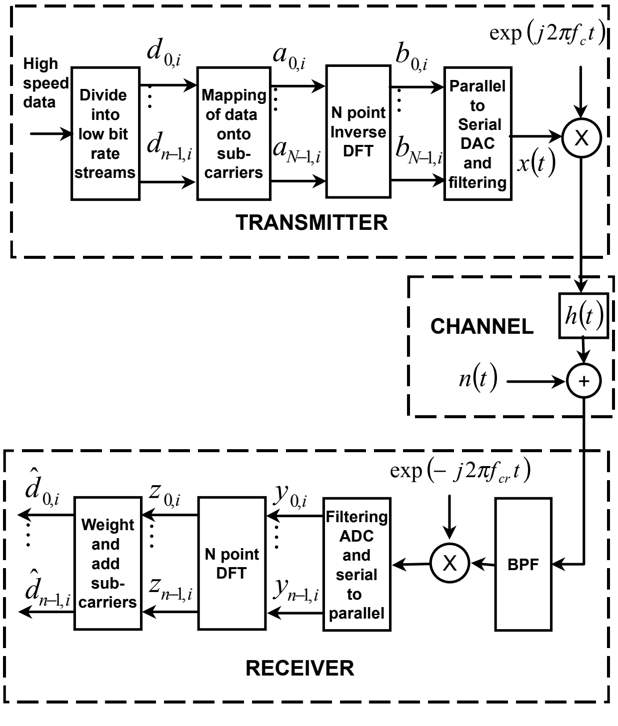

Figure 1: Structure of an OFDM communication system

When groups of three subcarriers are modulated, the relative weightings of the subcarriers are +1, -2, +1. In the general case for groups of k subcarriers the relative weightings are given by the coefficients of the polynomial $(1-x)^{k-1}$.

INTERCARRIER INTERFERENCE DUE TO FREQUENCY OFFSET

In OFDM a difference between the received carrier frequency and the receiver local oscillator frequency results in ICI. This frequency offset, Δf, can be caused either by a difference between the frequencies of the transmitter and receiver local oscillators, that is $f_{cr} \neq f_c$, or because of Doppler shift. The ICI resulting from frequency offset in an OFDM system can be analyzed in terms of complex weighting coefficients, $c_{0,i} \cdots c_{N-1,i}$ [3]. These coefficients give the contribution of each input to the given output. The outputs of the receiver FFT are given by

$$z_{m,i} = \exp(j\theta)\sum_{l=0}^{N-1} c_{l-m} a_{l,i} \tag{1}$$

where θ is the carrier phase error at the start of the received symbol period and

$$c_{l-m} = \frac{1}{N}\sum_{k=0}^{N-1} \exp\left(\frac{j2\pi k(l-m+\Delta fT)}{N}\right). \tag{2}$$

The weighting coefficient c_{l-m} gives the contribution of the l-th input a_l to the m-th output z_m. Each weighting coefficient, c_{l-m}, depends on the normalized frequency offset, ΔfT, and on $l-m$, but does not depend directly on m.

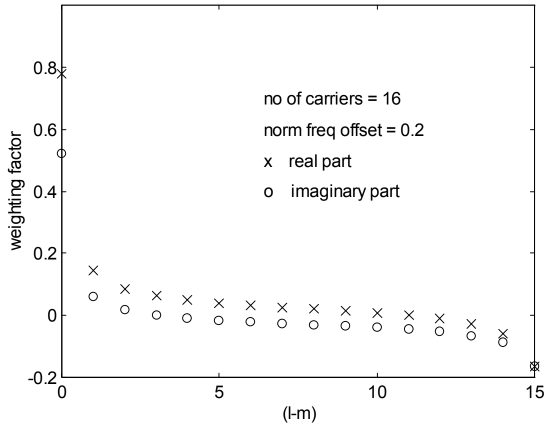

Figure 2: Real and imaginary parts of the complex weighting coefficients for $N=16$, and $\Delta fT = 0.2$.

Figure 2 shows the real and imaginary parts of the weighting coefficients for the case of $N=16$, and $\Delta fT = 0.2$. Figure 3 shows, in the form of phasor diagrams, the ICI caused by this frequency offset when a_2 is the only non-zero input.

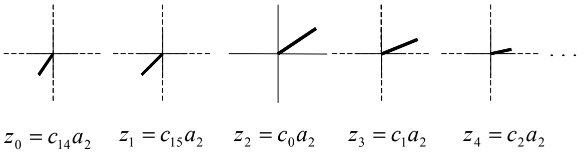

Figure 3: Values of first five DFT outputs, due to the single input a_2 with $N=16$, and $\Delta fT = 0.2$ (Note scale change by a factor of 5 between z_2 and other graphs)

The relationship between adjacent coefficients is more obvious if (2) is simplified by using the formula for the sum of a geometric progression. After some manipulation, it can be shown that the weighting coefficients are given by

$$c_{l-m} = \frac{1}{N}\frac{\sin\pi(l-m+\Delta fT)}{\sin\pi\left(\frac{l-m+\Delta fT}{N}\right)} \exp j\pi\left(\frac{(l-m+\Delta fT)(N-1)}{N}\right) \tag{3}$$

From this it can be seen that the phase of each coefficient is given by

$$\angle c_{l-m} = \frac{\pi\Delta fT(N-1)}{N} - \frac{\pi(l-m)}{N} \tag{4}$$

c_0 is the weighting coefficient which relates a given transmitted value of a_k to the corresponding value of z_k in the receiver. Thus the wanted signal in each subcarrier is phase rotated by $\angle c_0 = \pi\Delta fT(N-1)/N$. There is a constant phase shift of $-\pi/N$ between adjacent weighting coefficients.

It can be shown [6] that the total ICI resulting from a number of subcarriers can be modelled as Gaussian noise. However Figures 2 and 3 show that the ICI resulting from a single subcarrier is far from random in form. The component of ICI in one subcarrier resulting from the input a_2 is quite similar in value to the component in adjacent subcarriers. This fact is used in PCC. For PCC with $k=2$, adjacent pairs of subcarriers are modulated with opposite values, so that the resulting components of ICI in each of the other subcarriers tend to cancel. To generate the best estimates of the transmitted data, corresponding pairs of $z_{0,i} \cdots z_{N-1,i}$ should be subtracted to calculate $\hat{d}_{0,i} \cdots \hat{d}_{n-1,i}$ and this gives further ICI cancellation.

Figure 4 shows the signal to ICI ratio as a function of normalized frequency offset, ΔfT , for normal OFDM, and for PCC where groups of two and three subcarriers are weighted in the transmitter and the corresponding groups are weighted and combined in the receiver. The figure shows the case for $N = 16$, but the graphs for any $N \geq 8$ would have almost identical values [3].

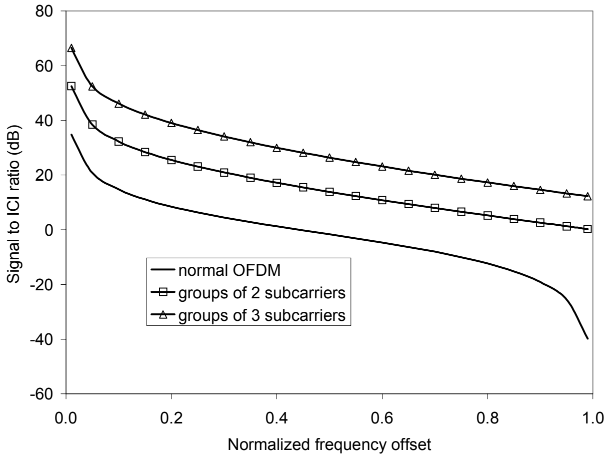

Figure 4: Effect of frequency offset. Wanted signal power/uncancelled ICI power as a function of normalized frequency offset, ΔfT .

IMPAIRMENTS CAUSED BY MULTIPATH PROPAGATION

The behaviour of the system shown in Figure 1 will be analysed for the case of multipath propagation. The analysis will not include the effect of additive noise in the channel. In a real OFDM system the transmitter output signal depends on the practical details of the DAC and the low pass filtering. In this analysis we will consider the case of 'ideal' OFDM, that is we will assume that the output of the transmitter is the sum of sinusoidal tone bursts of length *T*.

$$x(t) = \exp(j2\pi f_c t)\sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi lt}{T}\right), \quad \text{for} \quad (i-1)T < t < iT . \tag{5}$$

Ideally, there is no distortion or added noise in the channel, the receiver local oscillator has exactly the correct phase and frequency, and there is perfect symbol synchronization in the receiver. In this case, $y_{0,i} \cdots y_{N-1,i} = b_{0,i} \cdots b_{N-1,i}$ and $z_{0,i} \cdots z_{N-1,i} = a_{0,i} \cdots a_{N-1,i}$: the data is perfectly recovered.

However, in a multipath channel, a number of echoes of the transmitted signal are received, each echo subject to different delay and Doppler shift. This causes distortion of the received analogue signal and as a result noise and distortion in the decoded values, $z_{0,i} \cdots z_{N-1,i}$. We will first consider the effect of delay alone.

Impairments due to delay

In general, delay has a number of adverse effects on the received signal. It causes phase rotation so that each of the decoded values $z_{0,i} \cdots z_{N-1,i}$ is rotated with respect to $a_{0,i} \cdots a_{N-1,i}$. If the delayed echoes overlap subsequent symbols, delay can also cause intersymbol interference. Finally delay can cause ICI within a symbol. These latter two effects, can be eliminated by adding to each transmitted symbol a cyclic prefix, longer than the channel impulse response. [1]. In the following analysis we assume that a cyclic prefix, longer than the impulse response of the channel has been added.

Let the received signal due to the *p*-th transmission path be

$$\begin{aligned} v_p(t) &= g_p \exp j2\pi f_c(t-\tau_p)\sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi l(t-\tau_p)}{T}\right) \\ &= g_p \exp j(2\pi f_c t + \theta_p)\sum_{l=0}^{N-1} a_{l,i} \exp j\left(\frac{2\pi lt}{T} + l\phi_p\right) \\ &= \exp(j2\pi f_c t)\sum_{l=0}^{N-1} g_p a_{l,i} \exp\left(j\frac{2\pi lt}{T}\right)\exp j(\theta_p + l\phi_p) \end{aligned},$$

$$\text{for} \quad (i-1)T < t < iT \tag{6}$$

where g_p is the gain of this path and τ_p is the delay.

By inspection, the component of $z_{l,i}$ due to this echo is given by $g_p a_{l,i} \exp(j(\theta_p + l\phi_p))$. This is an attenuated, phase rotated version of the transmitted complex value.

The phase rotation is the sum of two values, θ_p and $l\phi_p$, where $\theta_p = -2\pi f_c \tau_p$, which is the same for all subcarriers and $l\phi_p = -l2\pi\tau_p/T$ which is a linear function of subcarrier frequency. For typical systems subject to multipath, the impulse response of the channel will be much longer than one period at the carrier frequency and so the values of θ_p due to different echoes will be approximately uniformly distributed over 2π radians. The components due to different echoes add vectorially so that, depending on the relative phasing of the components, the overall amplitude of the decoded subcarrier will increase or decrease. In other words fading will occur. In cases where the symbol period is much longer than the impulse response of the channel, $l\phi_p$ is very small, and all the subcarriers in a symbol are subject to the same attenuation and rotation. This is the flat fading case.

Impairments due to Doppler shift

When an echo is subject to Doppler shift, the received signal is compressed, or extended, in time compared with the transmitted signal. With OFDM this changes both the frequency of the carrier and the frequencies of each of the subcarriers. For typical values of Doppler shift, the change in each of the subcarrier frequencies is negligible and this will be ignored in the following analysis. The effect of Doppler shift can therefore be calculated using the analysis for carrier frequency offset.

Received signal due to one delayed, Doppler shifted echo

Consider the received signal due to one path. If g_p is the gain, τ_p is the delay and Δf_p is the Doppler shift for this path, then the received signal due to this path is

$$\begin{aligned} v_p(t) &= g_p \exp j2\pi\left(f_c + \Delta f_p\right)\left(t - \tau_p\right) \\ &\quad \sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi l\left(t-\tau_p\right)}{T}\right) \\ &= g_p \exp j\left(2\pi\left(f_c + \Delta f_p\right)t + \theta_p\right) \\ &\quad \sum_{l=0}^{N-1} a_{l,i} \exp j\left(\frac{2\pi lt}{T} + l\phi_p\right) \\ &= \exp j2\pi\left(f_c + \Delta f_p\right)t \\ &\quad \sum_{l=0}^{N-1} g_p a_{l,i} \exp\left(j\frac{2\pi lt}{T}\right) \exp j\left(\theta_p + l\phi_p\right) \end{aligned} \tag{7}$$

for $(i-1)T < t < iT$

where in this case $\theta_p = -2\pi\left(f_c + \Delta f_p\right)\tau_p$

Thus the component of $z_{m,i}$ due to this path is given by

$$z_{m,i,p} = g_p \exp\left(j\theta_p\right)\sum_{l=0}^{N-1} c_{l-m,p} a_{l,i} \exp\left(jl\phi_p\right), \tag{8}$$

where the complex weighting coefficients are for a frequency offset of Δf_p. For the flat fading case this can be simplified to

$$z_{m,i,p} = g_p \exp\left(j\theta_p\right)\sum_{l=0}^{N-1} c_{l-m,p} a_{l,i}\,. \tag{9}$$

OVERALL PERFORMANCE IN A MULTIPATH CHANNEL

The overall performance in a given multipath channel depends on the signal-to-interference ratio in the values $\hat{d}_{0,i} \cdots \hat{d}_{n-1,i}$. This in turn depends on how the components due to the signals with different propagation paths and resulting from different transmitted subcarriers combine. Performance also depends on whether the ICI increases and decreases as the desired signal increases and decreases, and whether these variables are uncorrelated. Such a calculation is beyond the scope of this paper. Here we will calculate values for the average power in the wanted component of each $\hat{d}_{0,i} \cdots \hat{d}_{n-1,i}$ and the average ICI power in each, where the average is over all possible transmitted data sequences. Calculations will be made for both ordinary OFDM and PCC OFDM.

Combination of different multipath components for normal OFDM

For normal OFDM the decoded values are simply the individual outputs of the receiver FFT, $z_{0,i} \cdots z_{N-1,i}$. Using the flat fading assumption discussed earlier, the different echoes arrive with carrier phase randomly spread over 2π radians. Thus the average total power of the received signal and hence the average power of each of the decoded values $z_{0,i} \cdots z_{N-1,i}$ can be found by summing the average power in each echo.

$$\begin{aligned} E\left[\left|z_{m,i}\right|^2\right] &= \sum_{paths} E\left[\left|z_{m,i,p}\right|^2\right] \\ &= \sum_{paths} E\left[\left|g_p \exp\left(j\theta_p\right)\sum_{l=0}^{N-1} c_{l-m,p} a_{l,i}\right|^2\right] \\ &= \sum_{paths} g_p{}^2 E\left[\exp\left(j\theta_p\right)\left|\sum_{l=0}^{N-1} c_{l-m,p} a_{l,i}\right|^2\right] \end{aligned} \tag{10}$$

Assuming that there is no correlation between the phase of a given path and the Doppler shift in that path, this can be simplified to

$$E\left[\left|z_{m,i}\right|^2\right] = \sum_{paths} g_p^{\,2} E\left[\left|\sum_{l=0}^{N-1} c_{l-m,p}\, a_{l,i}\right|^2\right]. \qquad (11)$$

This can be further simplified if the data modulating different subcarriers are zero mean, identically distributed independent random variables so that

$E\left[a_{l,i} a_{k,i}\right] = 0$ for $l \neq k$ and $E\left[\left|a_{l,i}\right|^2\right] = E\left[\left|a\right|^2\right]$. Then

$$\begin{aligned} E\left[\left|z_{m,i}\right|^2\right] &= \sum_{paths} g_p^{\,2} \sum_{l=0}^{N-1} \left|c_{l-m,p}\right|^2 E\left[\left|a_{l,i}\right|^2\right] \\ &= E\left[\left|a\right|^2\right] \sum_{paths} g_p^{\,2} \sum_{l=0}^{N-1} \left|c_{l-m,p}\right|^2 \end{aligned} \qquad (12)$$

Separating out the terms that are due to the wanted subcarrier and the terms which represent ICI gives

$$\begin{aligned} E\left[\left|z_{m,i}\right|^2\right] &= E\left[\left|a\right|^2\right] \sum_{paths} g_p^{\,2} \left|c_{0,p}\right|^2 \\ &\quad + E\left[\left|a\right|^2\right] \sum_{paths} g_p^{\,2} \sum_{l \neq 0} \left|c_{l,p}\right|^2 \end{aligned} . \qquad (13)$$

Thus for normal OFDM in a multipath environment the average wanted signal power to average ICI power is given by

$$\frac{\text{average wanted signal power}}{\text{average ICI power}} = \frac{\sum_{paths} g_p^{\,2} \left|c_{0,p}\right|^2}{\sum_{paths} g_p^{\,2} \sum_{l \neq 0} \left|c_{l,p}\right|^2} . \qquad (14)$$

Combination of different multipath components for PCC OFDM

In PCC OFDM the values of $d_{0,i} \cdots d_{n-1,i}$ rather than $a_{0,i} \cdots a_{N-1,i}$ are independent random variables, so first express the component of one output due to one multipath in terms of $d_{0,i} \cdots d_{n-1,i}$. We will consider the case of coding in pairs so that $a_{0,i} = d_{0,i} = -a_{1,i}$ and $\hat{d}_{0,i} = z_{0,i} - z_{1,i}$. Using this in conjunction with equation (8) and after some manipulation gives

$$\begin{aligned} \hat{d}_{m,i,p} &= g_p \exp\left(j\theta_p\right) \\ &\quad \sum_{k=0}^{N/2-1} \left(-c_{2k-2m-1,p} + 2c_{2k-2m,p} - c_{2k-2m+1,p}\right) d_{k,i} \end{aligned} . \qquad (15)$$

Again by assuming that components from different paths are arriving with random phases, that there is no correlation between the phase and the Doppler shift, and that the input data values $d_{0,i} \cdots d_{n-1,i}$ are zero mean, identically distributed independent random variables so that $E\left[d_{l,i} d_{k,i}\right] = 0$ for $l \neq k$, and $E\left[\left|d_{l,i}\right|^2\right] = E\left[\left|d\right|^2\right]$. Then after some manipulation it can be shown that

$$\begin{aligned} E\left[\left|d_{m,i}\right|^2\right] &= E\left[\left|d\right|^2\right] \sum_{paths} g_p^{\,2} \left|-c_{-1,p} + 2c_{0,p} - c_{1,p}\right|^2 \\ &\quad + E\left[\left|d\right|^2\right] \sum_{paths} g_p^{\,2} \sum_{k \neq 0} \left|-c_{2k-1,p} + 2c_{2k,p} - c_{2k+1,p}\right|^2 \end{aligned} \qquad (16)$$

Thus for PCC OFDM in a multipath environment, with coding onto pairs of subcarriers the average wanted signal power to average ICI power is given by

$$\frac{\text{average wanted signal power}}{\text{average ICI power}} = \frac{\sum_{paths} g_p^{\,2} \left|c_{-1,p} + 2c_{0,p} - c_{1,p}\right|^2}{\sum_{paths} g_p^{\,2} \sum_{k \neq 0} \left|-c_{2k-1,p} + 2c_{2k,p} - c_{2k+1,p}\right|^2} \qquad (17)$$

SIGNAL TO ICI POWER FOR CLASSICAL DOPPLER SPREAD

A well known and mathematically tractable model for a mobile channel is the classical Doppler spread model. Here the relative magnitudes of signals, as a function of frequency shift, is described by the classical Doppler spectrum [7],

$$S(v) = \frac{1}{\pi f_d \sqrt{1 - \left(\dfrac{v}{f_d}\right)^2}} , \qquad (18)$$

where v is the frequency shift and f_d is the maximum Doppler shift.

To calculate the signal to ICI for the classical Doppler case, the signal was considered as a large number of multipath components with random phases. The relative gain for a given frequency shift given by equation (18) and the signal to ICI for each path given by equation (14) for normal OFDM and equation (17) for PCC OFDM with pairs of subcarriers. A similar expression was derived for the case of groups of three subcarriers.

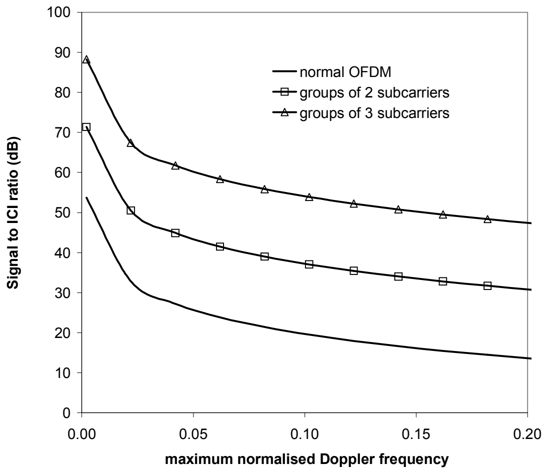

Figure 5 Effect of Doppler spread. Wanted signal power/uncancelled ICI power as a function of $f_d T$, the normalized maximum Doppler frequency.

Figure 5 shows the results for a classical Doppler spread channel, for normal OFDM and for PCC OFDM with groups of two and three weighted subcarriers. The plot is against $f_d T$, the maximum normalized Doppler frequency. The PCC with pairs of subcarriers give an improvement in signal to ICI ratio of between 10 and 20 dB depending on the maximum Doppler spread. Using groups of three subcarriers gives another 10-20dB of improvement.

CONCLUSIONS

The impairments caused by multipath propagation in PCC OFDM and ordinary OFDM have been analysed in detail. An expression has been derived for each output subcarrier in terms of each input subcarrier, and the gain, delay and complex weighting coefficients for each transmission path. By summing the components due to each transmission path the overall performance of PCC OFDM and ordinary OFDM for a given channel can be found. Calculations have been made for the classical Doppler spread channel model. It is shown that PCC OFDM has a much better signal to ICI power than ordinary OFDM. For PCC OFDM with mapping onto pairs of subcarriers there is an improvement of between 10-20dB, and for mapping onto groups of three carriers of between 20-40dB.

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