AMPLITUDE STATISTICS FOR POLYNOMIAL CANCELLATION CODED OFDM SIGNALS

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ABSTRACT

Polynomial cancellation coding (PCC) is a method of mapping data onto the subcarriers of OFDM which results in reduced sensitivity to frequency offset. [1] – [4]. In this paper it is shown that the amplitude statistics for PCC OFDM are different from ordinary OFDM and the statistics are not Gaussian. The results of simulations for OFDM and PCC OFDM are presented. It is shown that the signal statistics of PCC OFDM can be substantially improved by using a form of offset keying in which low index and high index subcarriers have different symbol periods.

1. INTRODUCTION

OFDM has become the modulation method of choice in many high speed data applications. In OFDM the single wideband transmission is replaced by many parallel narrowband transmissions [5,6]. Despite its many advantages, OFDM has two major disadvantages: its extreme sensitivity to frequency offset and its high peak to mean amplitude.

In earlier papers it was shown that the sensitivity to frequency offset could be very substantially reduced by using polynomial cancellation coding (PCC) [1,4]. It has also been shown that PCC OFDM has a better power spectrum and that the intersymbol interference (ISI) due to multipath transmission is much less in PCC OFDM than ordinary OFDM [2]. Despite these considerable advantages, PCC OFDM has also some drawbacks.

In this paper it is shown that the statistics of signal amplitude of PCC OFDM are not Gaussian. With PCC OFDM high amplitude signals occur more often than in normal OFDM. The results of simulations of PCC OFDM are presented. A new method of improving the amplitude statistics of PCC OFDM using a form of offset keying is described and analysed

2. DESCRIPTION OF PCC OFDM

Figure 1 shows the block diagram of an OFDM transmitter. This diagram is general enough to describe ordinary OFDM and PCC OFDM. The high-speed data to be transmitted is divided into n lower speed parallel channels. The data in the k-th parallel channel in the i-th symbol period is represented by $d_{k,i}$. This will in general be a complex value. The data values $d_{0,i} \cdots d_{n-1,i}$ are mapped onto the values $a_{0,i} \cdots a_{N-1,i}$ which modulate the N subcarriers in the i-th symbol period. For normal OFDM, $n = N$ and $a_{k,i} = d_{k,i}$; there is a simple one-to-one mapping of data onto the subcarriers. With PCC, the data to be transmitted is mapped onto weighted groups of subcarriers. For example, to apply PCC to pairs of subcarriers, the subcarriers in each pair must have relative weightings +1 and –1 [1]. In this case $n = N/2$. The first data value in each symbol period is mapped onto the first two subcarriers: $a_{0,i} = d_{0,i}$, $a_{1,i} = -d_{0,i}$. The second data value is mapped onto the third and fourth subcarrier and so on.

When groups of three subcarriers are modulated the relative weightings of the subcarriers are +1, -2, +1. In the general case for groups of m subcarriers the relative weightings are given by the coefficients of the polynomial $(1-x)^{m-1}$ [1].

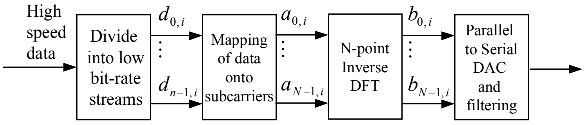


Figure 1: Block diagram of an OFDM transmitter

PCC also results in a form of windowing of the transmitted signals. Some forms of PCC can be implemented in the time, rather than frequency domain, but the window functions are different from those previously described in the literature and are in general complex [2]. The envelope of each signal peaks in the middle of the symbol period and falls to zero at each end. Figure 2 shows typical time domain signals for ordinary OFDM and PCC OFDM. Two symbol periods are shown in each case. For normal OFDM (Figure 2 (a)) the envelope is a constant, it is not a function of time. Figure 2(b) shows a typical time domain signal for PCC with mapping onto pairs of subcarriers. The envelope peaks in the middle of each symbol period and falls to zero at the end. Figure 2(c) shows that with mapping onto groups of three subcarriers the energy is even more concentrated in the centre of each symbol period.

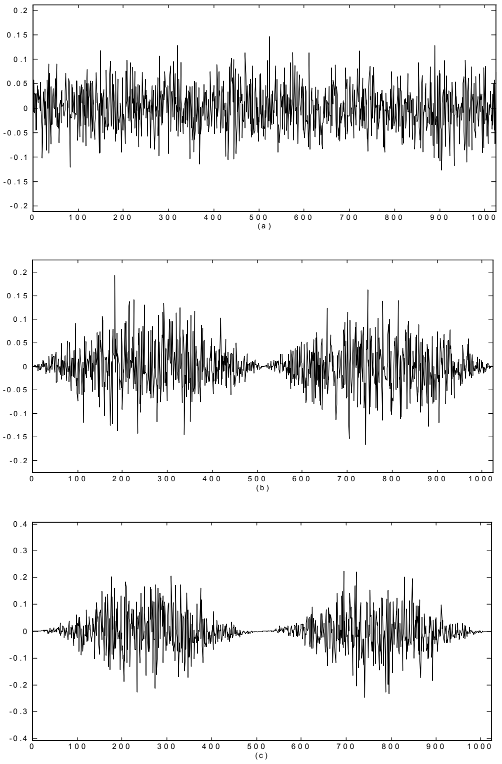


Figure 2: Time domain signals for OFDM:
(a) Normal OFDM; (b) PCC OFDM with mapping into pairs of subcarriers; (c) PCC OFDM with mapping into groups of three subcarriers

In the receiver the data is estimated by weighting and summing at the FFT output the members of each group of subcarriers. The same relative weightings are used as in the transmitter. This is a form of matched filtering. The receiver is matched to the signal carried by each weighted group. This weighting and adding process also gives polynomial cancellation properties of a higher degree which further reduces the sensitivity to frequency offset.

3. STATISTICS OF SIGNAL AMPLITUDE

A well known drawback of OFDM is the high peak to mean envelope power ratio [7]. Hence, if clipping is to be avoided, inefficient linear amplifiers are required. A number of methods of reducing this problem, such as block coding schemes have been described in the literature [8], [9].

The statistics of signal amplitude for OFDM are approximately Gaussian. Figure 3 shows the amplitude statistics resulting from the simulation of an OFDM transmitter with $N = 512$. The simulation was for 32 symbols. Also plotted is a Gaussian distribution of the same variance. The figure shows that the actual distribution is very accurately approximated by a Gaussian distribution.

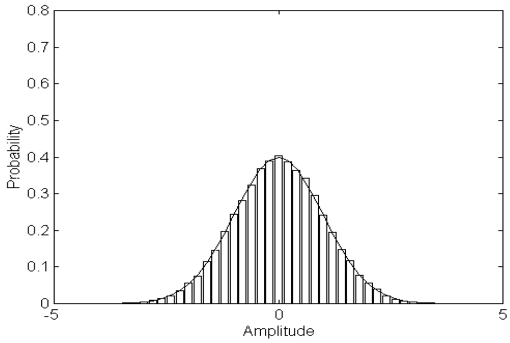


Figure 3: Amplitude statistics for OFDM signal compared with Gaussian distribution

Figure 4 shows the same case for PCC OFDM using pairs of subcarriers. For PCC the statistics are no longer Gaussian. For a given signal power both high and low amplitude signals occur with higher probability than for normal OFDM. This is because of the way the envelope peaks in the middle of each symbol period and falls to zero at each end. High amplitude signals occur with high probability in the middle of each symbol and near zero amplitude signals at each end.

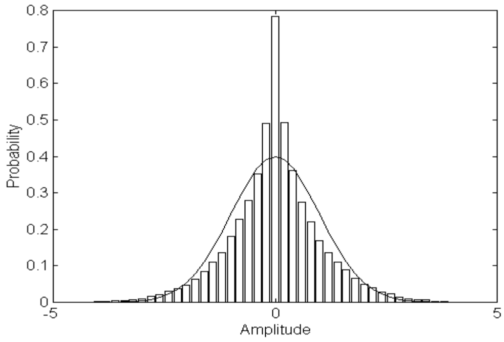


Figure 4: Amplitude statistics for PCC OFDM signal with mapping onto pairs of subcarriers compared with Gaussian distribution

4. DISTORTION DUE TO CLIPPING

One technique for eliminating the high amplitude peaks in OFDM signals is simply to clip them in the digital part of the transmitter so that they never reach the analogue amplifiers. This clipping results in a loss of energy in the transmitted signal and appears as added noise in the received signal. Simulations were performed to compare the loss in energy which would result from clipping at a given signal amplitude.

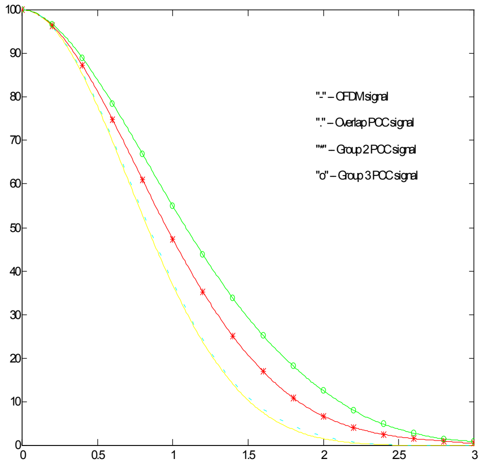


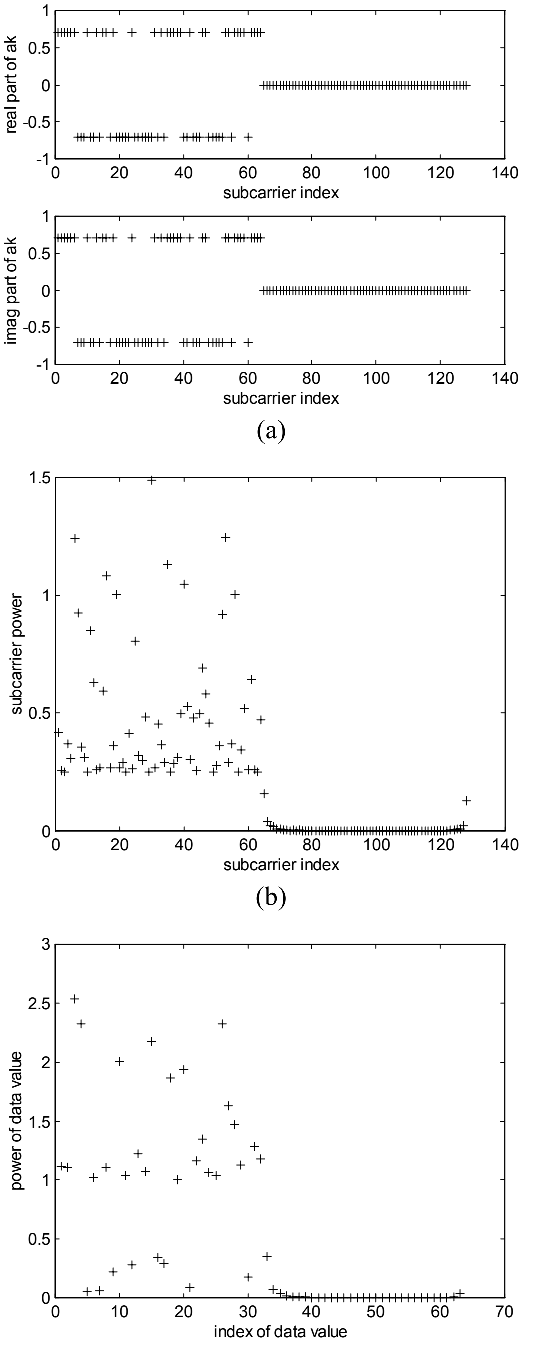
Figure 5: Power lost due to clipping

Figure 5 compares the power lost for a given clipping level for ordinary OFDM and for PCC OFDM. In practice, the signals would only be clipped at high amplitudes. For a clipping amplitude of 2, PCC OFDM with mapping onto pairs of subcarriers gives a significantly greater power loss than normal OFDM. For mapping onto groups of three this power loss is very large.

5. PCC WITH OFFSET KEYING

The time domain signal of a PCC OFDM signal peaks in the middle of each symbol period. As a result one way to improve the amplitude statistics of PCC OFDM while retaining the benefits such as reduced sensitivity to frequency offset, is to use a form of offset keying. The low index subcarriers are transmitted with symbol period 0 to T, T to $2T$ etc while the high index subcarriers are transmitted with symbol periods $T/2$ to $3T/2$, $3T/2$ to $5T/2$ etc. This results in the peaks in the time domain envelope of the signal due to lower subcarriers occurring at the same times as the nulls of the signal due to the higher subcarriers and vice versa.

For offset keying to be useful, it is essential that the original data can be recovered from the received signal with little or no distortion. This form of offset keying would not give acceptable performance in normal OFDM as the time domain overlap in symbols would cause a high level of intercarrier interference (ICI) in the demodulated subcarriers. However the polynomial cancellation properties of PCC that were originally developed to cancel ICI due to frequency offset, also result in cancellation of ICI due to the symbol overlap. The overlap results in substantial ICI only in the immediately adjacent subcarriers and so offset keying can be used if only one or two subcarriers between the lower and higher subcarriers are left unused.



(a)

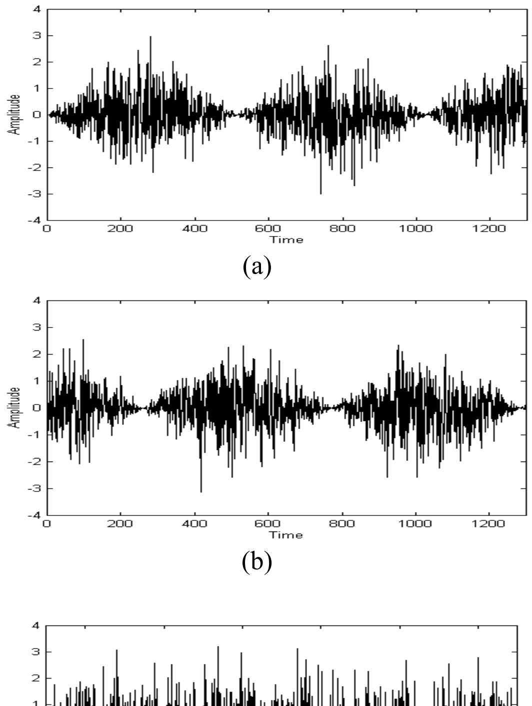
(b)

(c)

Figure 6: Intercarrier interference in subcarriers $N/2$ to $N-1$ caused by subcarriers 0 to $N/2$-1, with offset in symbol period of $T/2$ (a) input to transmitter IDFT, (b) output from receiver DFT, (c) result of weighting and summing receiver DFT outputs

Figure 6 demonstrates the ICI that would result for this form of offset keying. Figure 6(a) shows the signal being input to the inverse DFT in the OFDM transmitter. The first $N/2$ subcarriers are QAM with random data. The power of each subcarrier is unity. PCC coding onto pairs of subcarriers has been used.

The second $N/2$ subcarriers have zero amplitude. Figure 6(b) shows the resulting output of the receiver DFT if the received signal is decoded with a time offset of half a symbol period. The graph shows the power in each subcarrier. Figure 6(c) shows the output after groups of subcarriers have been weighted and summed. This shows what ICI would result in the demodulated output from this form of offset keying. The decoded signal in the first $N/2$ subcarriers is not of importance as these values would not be used in decoding the data. The decoded signal in the second $N/2$ subcarriers represents ICI. Comparing figures 6(b) and 6(c) shows the extra ICI cancellation which results from the weighting and summing process. It can be seen that the ICI is only significant in a few demodulated outputs.



(a)

(b)

(c)

Figure 7: Time domain signals for PCC with offset keying. (a) time domain signal as a result of lower index subcarriers, (b) time domain signal as result of higher index subcarriers, (c) resultant time domain signal.

Figure 7 shows the results of a time domain simulation for PCC OFDM with offset keying. Figure 7(a) shows the component of the time domain signal due to the low index subcarriers, while figure 7(b) shows the component due to the high index subcarriers. Note the offset of $T/2$ in symbol period. Figure 7(c) shows the result of summing the signals in (a) and (b). There is no discernible windowing effect for the resulting signal.

Figure 8 shows the statistics of signal amplitude for PCC OFDM with this form of offset keying. Like ordinary OFDM the statistics are very closely approximated by a Gaussian distribution. Simulations were also performed on the loss of energy which would result if clipping were used to limit the signal amplitude. These results are plotted in figure 5. There is no significant difference in the energy loss due to clipping between ordinary OFDM and PCC OFDM with offset keying.

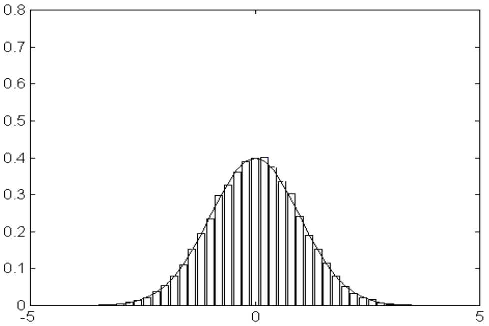


Figure 8: Amplitude statistics for PCC OFDM with offset keying

6. CONCLUSIONS

The results of simulations of PCC OFDM have been presented. It has been shown that the amplitude statistics of PCC OFDM are not Gaussian; low and high amplitude signals occur with greater probability. This is a major disadvantage which would result in impractical requirements for the transmitter output amplifiers and/or increased signal loss due to signal clipping. It has been shown that a simple method of improving the amplitude statistics of PCC OFDM while retaining the advantages of PCC OFDM is to use a form of offset keying. Results have been presented for this form of offset keying showing that the ICI which results is limited to a few subcarriers and that the amplitude statistics are approximately Gaussian. This form of offset keying is therefore a practical way of gaining the benefits of PCC OFDM while overcoming one of its disadvantages.

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