Frequency Domain Equalization for PCC-OFDM with overlapping symbol periods

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ABSTRACT

Polynomial cancellation coding (PCC) is a coding method for orthogonal frequency division multiplexing (OFDM) in which the information to be transmitted is modulated onto weighted groups of subcarriers rather than onto individual subcarriers. It has previously been shown that compared with ordinary OFDM, PCC-OFDM has very much reduced sensitivity to frequency offset and Doppler spread, lower out-of-band power and reduced intersymbol interference (ISI) due to multipath transmission. In its simplest form PCC-OFDM results in a reduction in overall bandwidth efficiency. One way of retaining the benefits of PCC-OFDM without loss of bandwidth efficiency is to overlap the transmitted symbols. At the receiver an equalizer is used to recover the data. The equalizers are two-dimensional. In this paper the results are presented for simulations of a number of different equalizer structures. It is shown that by using a decision feedback equalizer, the data can be recovered with approximately 1.5dB SNR degradation compared with the ideal case.

1. POLYNOMIAL CANCELLATION CODED OFDM SYSTEMS

OFDM is the modulation method chosen for many high-speed digital communication systems. This is despite OFDM having a number of well-known disadvantages that include extreme sensitivity to frequency offset, large out-of-band power, and high peak-to-mean power ratio. One of the often-quoted advantages of OFDM is that by using a cyclic prefix it can be made insensitive to multipath transmission. However this is at the cost of some loss in bandwidth efficiency. One technique which has the potential to solve many of the problems of OFDM is Polynomial Cancellation Coding (PCC) [1,2]. In PCC-OFDM the data to be transmitted is mapped onto weighted groups of subcarriers rather than individual subcarriers.

Figure 1.1 shows the block diagram of a PCC-OFDM communication system. Compared with standard OFDM there is an extra block in the transmitter to map the data onto the subcarriers. There is also an extra block in the receiver marked ‘weight and add subcarriers’. This combines the received subcarriers in a group and can be shown to result in matched filtering of the received signal [3]. The same weightings are used in the transmitter and receiver and these are chosen so that intercarrier interference (ICI) tends to cancel. The results are completely general and any number of subcarriers can be used in a group. In most practical situations, groups of only two subcarriers are required. In this case the two subcarriers in a group are given relative weightings +1 and –1.

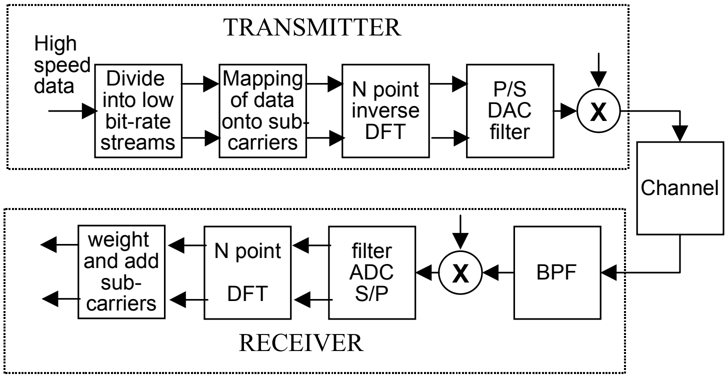

Fig. 1.1 Block diagram of PCC-OFDM communication system

PCC-OFDM has been shown to be much less sensitive to frequency offset and Doppler spread than standard OFDM [1,2]. It has also been shown to have better spectral roll-off and reduced sensitivity to multipath propagation [3]. These properties of PCC-OFDM can be understood by considering the result of weighted pairs of subcarriers in the frequency domain and in the time domain.

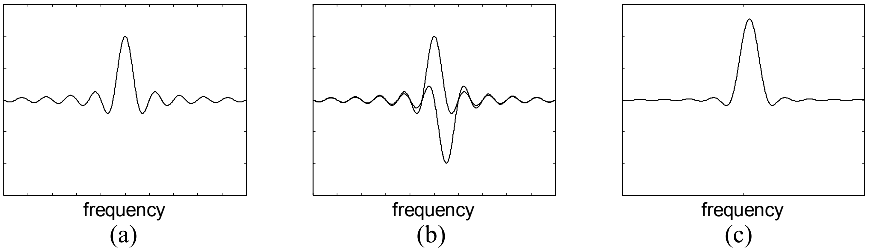

Fig. 1.2 PCC-OFDM in the frequency domain – spectra of subcarriers. (a) One subcarrier, (b) Two adjacent subcarriers, (c) Sum of weighted pair of subcarriers

In OFDM the spectrum of each subcarrier has a $\sin x/x$ form. Figure 1.2 shows how the weighting of a pair of subcarriers in PCC results in a canceling of the sidelobes and a very much faster overall spectral roll-off. As a result the overall power spectrum of a PCC-OFDM signal with mapping onto pairs of subcarriers falls off as $1/(N^3 f^4)$compared with $1/(Nf^2)$ for standard OFDM.

In the time domain the weighting and adding of pairs of subcarriers results in a windowing effect. The windowing function, which is equivalent to PCC-OFDM, is not identical to any of the windowing functions described in the literature. It is complex, in contrast to all windowing functions previously described, which are real.

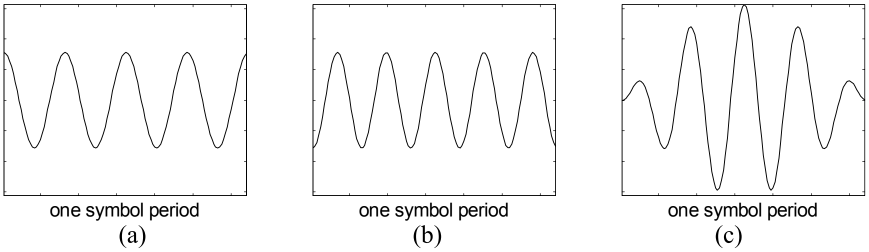

Fig. 1. 3 PCC-OFDM in the time domain – adding subcarriers results in sinusoidal envelope. (a) One subcarrier, (b) Next subcarrier with opposite polarity, (c) Sum of pair of subcarriers

Figure 1.3 shows the combination of the real components of two adjacent subcarriers with opposite weighting. Note the sinusoidal envelope. This results in most of the energy of a PCC-OFDM symbol being concentrated in the centre of the symbol period. This also means that PCC-OFDM is much less sensitive to ISI due to multipath transmission.

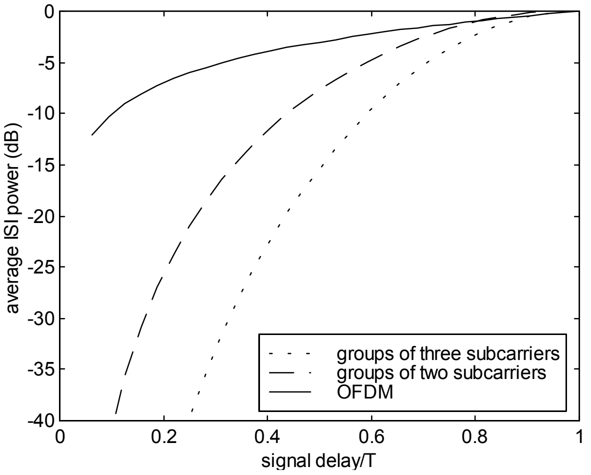

Fig. 1. 4 Power of ISI caused by multipath transmissions in OFDM and PCC-OFDM

Figure 1. 4 compares the power of ISI for normal OFDM and PCC-OFDM for the case of a matched filter receiver [3]. For signals delayed by 0.2*T*, a typical value used in the design OFDM systems, the multipath signal is reduced by approximately 20dB in the system using subcarrier pairs compared with normal OFDM. As a result, for many typical channels, PCC-OFDM does not require the cyclic prefix usually required for OFDM.

2. SPECTRAL EFFICIENCY OF PCC-OFDM

Despite its many advantages, PCC-OFDM used in its simplest form has one major disadvantage - loss in spectral efficiency. The use of two subcarriers, instead of one, to transmit each complex value, reduces the spectral efficiency by half. The elimination of the cyclic prefix, the improved spectral roll-off and the reduction in ICI will make up for some, but not all, of this loss.

Two ways of retaining the benefits of PCC-OFDM, while increasing the spectral efficiency have been explored. These can broadly be described as ‘overlapping in frequency’ and ‘overlapping in time’. It can be shown that ‘overlapping in frequency’ is a form of the well-known technique called partial-response signaling [4]. A number of groups have independently been researching this and similar approaches [5,6]. However our research has shown that it is not particularly effective. This is because the ‘weighting and adding’ block in the PCC-OFDM receiver contributes significantly to the improved performance [7] and this block can not readily be incorporated into a partial-response system.

The second approach, ‘overlapping in time’ in which the symbol periods of adjacent symbols overlap gives much better results. Figure 2.1 shows the basic idea of overlapping in the time domain.

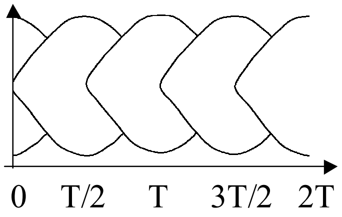

Fig. 2.1 PCC-OFDM with symbols overlapping in time domain

Symbols of duration T are transmitted at intervals of less than T, typically $T/2$. In other words intersymbol interference (ISI) is deliberately introduced at the transmitter to increase the data rate. For the case of an overlap of $T/2$, the transmitted signal at any instant is the sum of components from two symbols. This could be achieved by adding some extra circuitry in the transmitter after the inverse DFT.

Figure 2.2 shows a block diagram for the transmitter. The rate of clocking of the IDFT must also be increased to the new symbol rate.

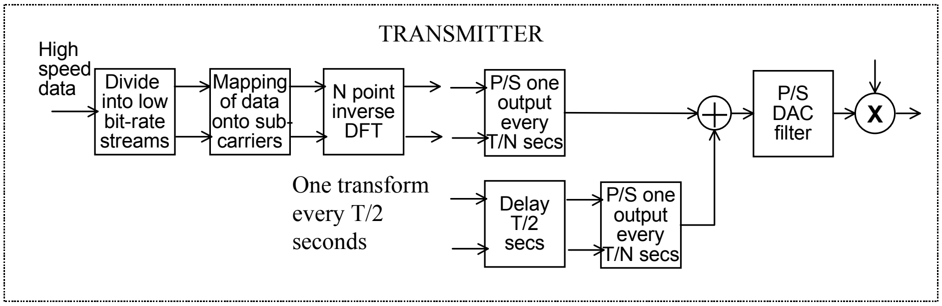

Fig. 2.2 OFDM transmitter with PCC and overlapping symbol periods

Figure 2.3 shows a block diagram for the receiver. An equalizer is required to recover the transmitted data. This can be either a time domain equalizer or a frequency domain equalizer but as will be shown particularly simple and effective frequency domain equalizers can be designed. The rate of clocking of the receiver DFT must also be increased to the new symbol rate. Figure 2.3 also shows the form of the two-dimensional frequency domain equalizer.

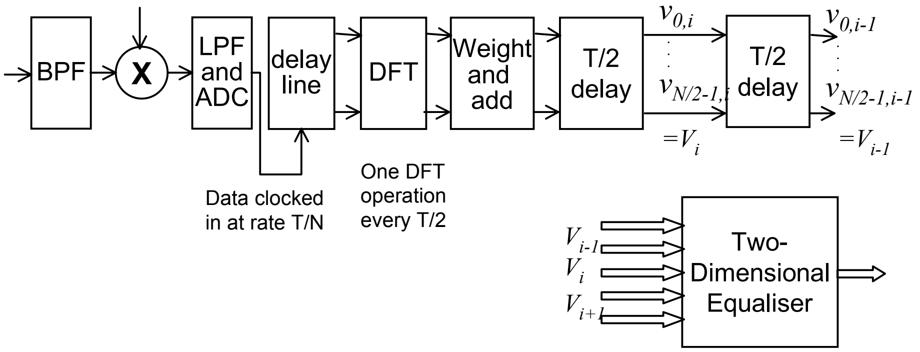

Fig. 2.3 OFDM receiver with PCC and overlapping symbol periods

One way of describing PCC-OFDM with overlapping symbol periods is as a new modulation scheme based on ‘almost but not quite’ orthogonal functions which are insensitive to time and frequency errors. In contrast, standard OFDM is based on functions which, in the absence of distortion, are absolutely orthogonal but which are very sensitive to these errors. The overlapping of symbol periods destroys the orthogonality, but because of the sinusoidal envelope of PCC-OFDM, this departure from orthogonality is quite small and results in an almost negligible loss in SNR. Partial-response signaling causes a much greater loss in orthogonality and a consequentially greater SNR penalty. Very recently two groups have reported the use of non-orthogonal functions in OFDM. Matheus et al. considered the use of Gaussian pulses [8]. Their analysis included consideration of possible equalizers. Kozek and Molisch considered the optimization of pulse shapes subject to time and frequency spreading [9].

3. FREQUENCY DOMAIN EQUALIZERS FOR PCC-OFDM WITH OVERLAPPING SYMBOL PERIODS

3.1 Impulse response of system

Frequency domain equalizers are not normally used in OFDM to counteract the effects of multipath transmission. Instead a cyclic prefix is used. This is because the ICI resulting from ISI is spread across a large number of subcarriers and an impractical number of equalizer taps would be required [10]. The polynomial cancellation properties of PCC also work to cancel the interference caused by ISI so that there are only a few significant terms. This means that the equalizer structures can be much simpler. In general the interference is two-dimensional; it is between symbols and also between subcarriers within a symbol. The equalizers, which are being considered for PCC-OFDM, are two dimensional frequency domain equalizers, which operate on the outputs of the ‘weighting and adding’ block.

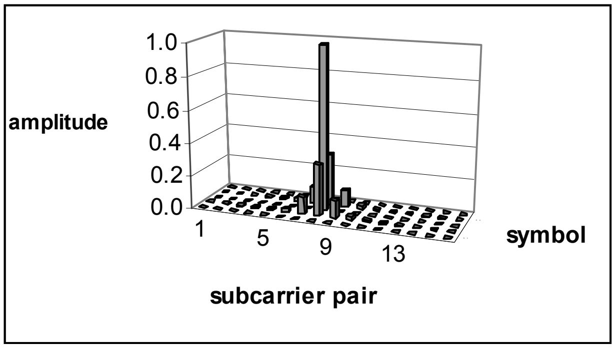

Fig 3.1 Amplitude of two dimensional impulse response for ideal channel

Figure 3.1 shows the outputs from the ‘weighting and adding’ block in the receiver, which result from one transmitted subcarrier pair in one symbol. It can be considered as a form of two-dimensional impulse response. Because each symbol overlaps in time, only with the preceding and following symbol, the impulse response has non-zero terms only for three symbols: the preceding, the current and the following symbol. In the current symbol, only the wanted subcarrier pair has a non-zero value. In the preceding and following symbols there are three significant terms; these are for the corresponding subcarrier pair and the subcarrier pairs on each side. The total impulse response has only seven terms greater than -20dB, only three of which are above -10dB. In a PCC-OFDM system with overlapping in time and an ideal channel the equalizer in the receiver would to have to equalize this response. For a real channel, the equalizer would also have to compensate for the added distortion in the channel. This would change the details of figure 3.1 but, because of the properties of PCC-OFDM, would still result in an impulse response with only a small number of significant terms.

A number of forms of equalizer have been simulated. The simulations were the simple case of an additive white Gaussian noise (AWGN) channel. To demonstrate the real potential of PCC-OFDM with overlapping symbol periods it will be necessary to consider more realistic channels that introduce significant ISI and ICI. This will be the subject of future work.

In all of the simulations OFDM with 32 subcarriers was used. Each subcarrier pair was modulated with 4QAM. The simulations were for 50000 transmitted symbols. In each case the performance is compared with that of normal OFDM. The results for OFDM are the same as those for any system using a number of orthogonal carriers and so are also identical to the performance of PCC-OFDM with no overlap.

3.2 Zero forcing linear equalizers

By extending well-known equalizer theory to two dimensions, linear zero forcing equalizers operating on three, five and seven symbols were designed. Figure 3.2 shows

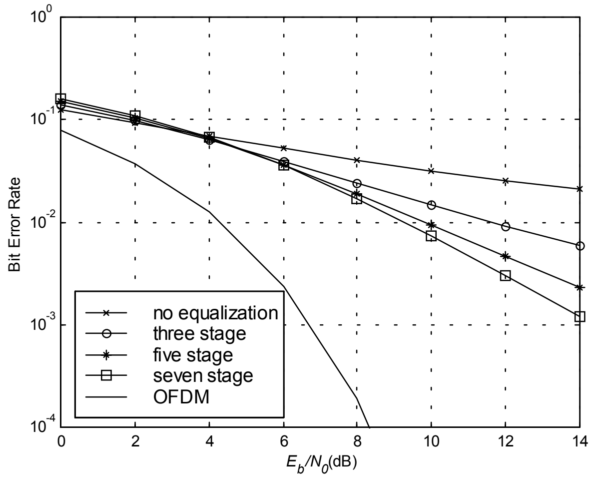

Fig. 3.2 BER for linear zero forcing equalizers.

the bit error rate (BER) as a function of E_b/N_0 for these equalizers. For comparison, the result for no equalizer and the result for normal OFDM with the same AWGN channel are also shown. The results for PCC-OFDM with no overlap are the same as for normal OFDM. The performance improves with the numbers of equalizer stages, but even for seven stages the performance is more than 3dB worse than that for normal OFDM. A better BER performance with the same bandwidth efficiency could be achieved using PCC-OFDM with no overlap, and increasing the size of the constellation.

3.3 Decision feedback equalization with zero forcing feedforward section

Figure 3.3 shows the results for decision feedback equalizers (DFEs) with a varying number of feedforward stages. In other words, decision feedback was used to subtract out the interference from the preceding symbol, linear equalization was used to reduce the interference from the following symbols. The feedforward stages were designed to meet the zero forcing criterion.

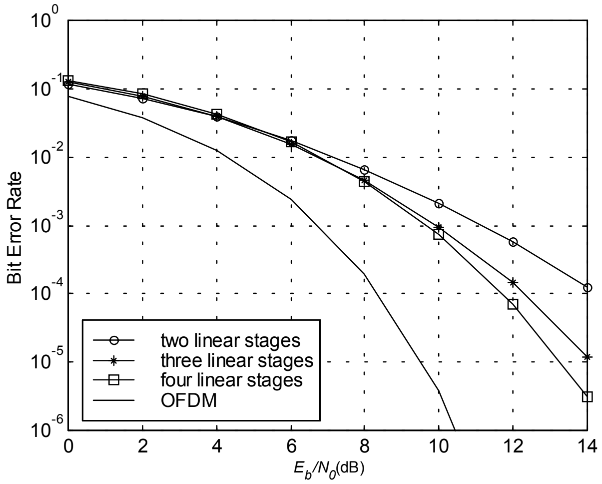

Fig. 3.3. BER for decision feedback equalizers with linear zero forcing feedforward section.

3.4 Decision feedback equalization with zero forcing feedforward section and error correction across a symbol

The results for the DFE are much better than for the linear equalizer, but the degradation compared with normal OFDM, or PCC-OFDM with no overlap is still more than 3dB. In this case the errors in decoding each subcarrier pair are being fed back and as will be shown, this contributes significantly to the BER. One way to reduce this error propagation is to use an error correcting code across each symbol. This would reduce significantly the probability of error for each subcarrier pair and hence the probability of error propagation. Figure 3.4 shows the results where the correct data values are used in the feedback. The performance is improved considerably and the BER is now within approximately 1.5dB of that for normal OFDM.

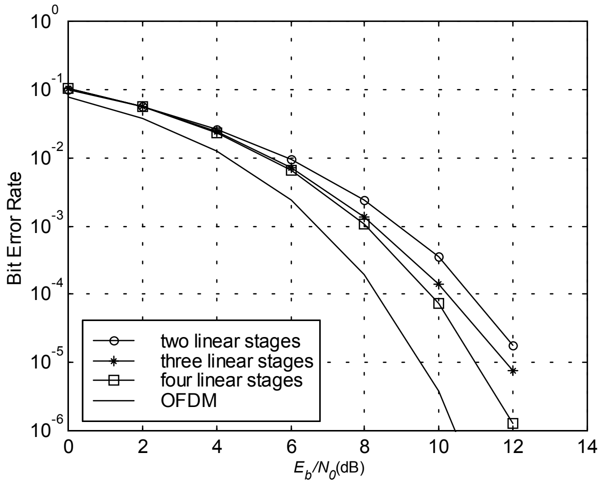

Fig. 3.4 BER for decision feedback equalizers with linear zero forcing feedforward section. Correct decisions fed back.

3.5 Other equalizer structures

By designing the feedforward section using the minimum mean square error criterion (MMSE) further improvements should be possible. The design of the MMSE section must take into account that the noise at the input to the equalizer is not white. The weighting and adding section introduces some correlation between the noise values in adjacent vectors.

Alternatively a maximum likelihood sequence estimator (MLSE) could be designed. Matheus [8] considered this approach for OFDM with Gaussian windowing but it is computationally complex. The properties of PCC-OFDM which concentrate the significant interference in only a few terms, and allow greater choice in the number of subcarriers, may lead to algorithms with significantly reduced complexity.

3.6 Equalization to counteract channel impairments

In any real application equalization would have to counteract the effect of channel impairments as well as correct for the deliberately introduced ISI. A number of papers have considered the use of vector equalization for OFDM [11,12]. The design of equalizers and the overall performance of PCC-OFDM for typical channels remain topics for further study. However the properties of PCC-OFDM which concentrate the interference in only a few terms should make vector equalization for this case much simpler than for normal OFDM.

4. CONCLUSIONS

PCC-OFDM is a new method of combining coding and modulation to improve the properties of OFDM. By overlapping PCC-OFDM symbols in time, these improvements can be gained while at the same time *increasing* the overall spectral efficiency of the OFDM system. Equalizers are required in the receiver to recover the transmitted data. In this paper the results of simulations of a PCC-OFDM with frequency domain equalizers have been presented. It has been shown that comparatively simple decision feedback equalizers can cancel the distortion with minimal noise enhancement. Zero forcing linear equalizers have been shown to give very poor results, as they are both ineffective at equalizing the signal and cause considerable noise enhancement. These results indicate that by using PCC-OFDM with overlapping symbol periods, systems can be designed for many applications including mobile telephony, digital television and ADSL with higher data rates and better performance than existing OFDM systems.

5. REFERENCES

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