

# New OFDM Peak-to-Average Power Reduction Scheme\*

Jean Armstrong

Department of Electronic Engineering, La Trobe University, Bundoora, Victoria 3086, Australia

Email: j.armstrong@ee.latrobe.edu.au

## Abstract

*This paper describes a new peak-to-average power ratio (PAPR) reduction scheme for orthogonal frequency division multiplexing (OFDM). A time domain version of the OFDM signal is generated using an oversized inverse discrete Fourier transform (DFT). This results in trigonometric interpolation. The interpolated signal is clipped. To remove the resulting out-of-band components the clipped signal is filtered through the use of a forward and inverse DFT. The filter passes the wanted in-band discrete frequencies while nulling the out-of-band components. The new scheme gives lower PAPR than other clipping techniques. Results are presented for the power spectral density and in-band distortion when the scheme is followed by a non-ideal amplifier. No change to the receiver is required so the scheme is compatible with existing communications standards.*

## 1. Introduction

One of the main disadvantages of Orthogonal Frequency Division Multiplexing (OFDM) is its high peak-to-average power ratio (PAPR). OFDM transmitters therefore require very linear output amplifiers with wide dynamic range. These are expensive and inefficient. Any amplifier non-linearity causes intermodulation products resulting in unwanted out-of-band power. Although the PAPR is very large for OFDM, high magnitude peaks occur relatively rarely and most of the transmitted power is concentrated in signals of low amplitude.

The simplest approach to reducing the PAPR of OFDM signals is to clip the high amplitude peaks. A variety of clipping techniques have been described in the literature [1-2]. Some clip the outputs of the inverse fast Fourier transform (IFFT) before interpolation. However the signal must be interpolated before analogue to digital conversion, and this will cause peak regrowth [3].

To avoid the problem of peak regrowth, the signal can be clipped after interpolation. However this causes very significant out-of-band power. Some papers have described clipping of the interpolated signal followed by

filtering [1]. The filters used have been complicated. Filtering also causes peak regrowth, although this is less than for the case of clipping before interpolation.

This paper describes a new PAPR reduction technique in which clipping of the interpolated signal is followed by a new form of filter. Simulations of the new technique including the interaction with a non-ideal amplifier are presented showing improved performance.

## 2. New PAPR Reduction Technique

Figure 1 shows the block diagram of the new PAPR reduction scheme [4]. The input vector  $a_0 \dots a_{N-1}$  is first converted from the frequency to the time domain using an oversize IFFT.  $N$  is the number of subcarriers in each OFDM symbol. For an oversampling factor of  $I_1$ , the input vector is extended by adding  $N(I_1 - 1)$  zeros in the middle of the vector. This results in trigonometric interpolation of the time domain signal [5]. Trigonometric interpolation gives perfect interpolation when the original signal consists of integral frequencies over the FFT window. This is the case for OFDM.

The input at the Nyquist frequency,  $N/2$  has been omitted, as the interpolation technique does not work for this value [5]. This is not a practical limitation as all applications of OFDM null this input and most do not use a number of adjacent subcarriers. The interpolated signal is then clipped. In this paper, hard-limiting is applied to the amplitude of the complex values at the IFFT output. However the technique is quite general and any other form of non-linearity can be applied. The clipping ratio,  $CR$  is defined as the ratio of the clipping level to the mean power of the unclipped baseband signal.

The clipping is followed by frequency domain filtering to reduce out-of-band power. The filter consists of two FFT operations. The forward FFT transforms the clipped signal back into the discrete frequency domain. The in-band discrete frequency components of the clipped signal  $c_0 \dots c_{N/2-1}, c_{N/2+1} \dots c_{N-1}$  are passed unchanged to the inputs of the second IFFT while the out-of-band

\* This work was supported in part by an Australian Research Council Grant

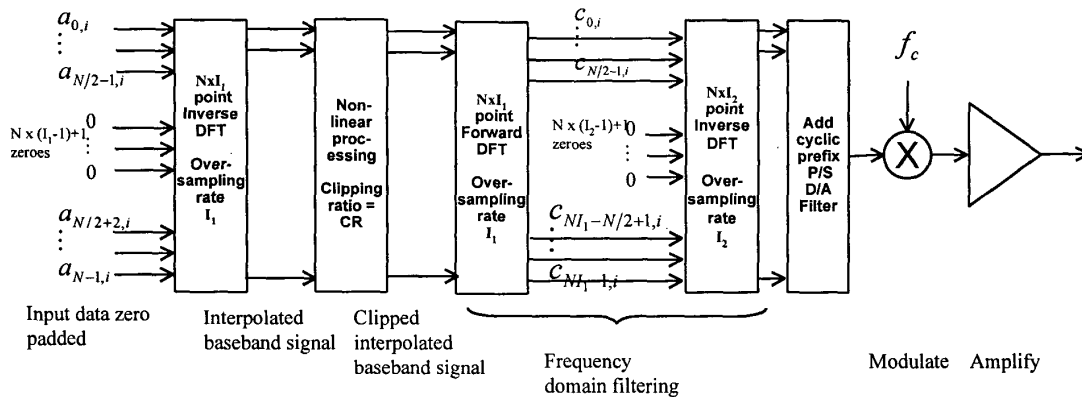


Figure 1. Block diagram of new peak reduction technique

components,  $c_{N/2+1} \dots c_{N-1-N/2}$  are nulled. The second IFFT is followed by serial/parallel conversion, digital to analogue conversion, modulation and amplification.

Although frequency domain filtering is a common signal processing technique the form shown in Fig. 1 is unusual. In most filtering applications the filter is designed to meet particular specifications in the *continuous* frequency domain. In this application, the wanted signal is an OFDM signal, which is the sum of discrete frequency components in each symbol period. The filter must therefore have as little effect as possible on the in-band *discrete* frequency domain while attenuating as much as possible any out-of-band components. This is precisely what is achieved by the simple filter structure in Fig. 1. Because the filter operates on a symbol by symbol basis there is no filtering across symbol boundaries and so no resultant intersymbol interference. The filtering does cause some peak regrowth. See Fig. 2. However this is much less than for clipping before interpolation.

Figure 3 shows the distribution of instantaneous signal power as a function of CR. The graphs show the power that a given percentage of signal samples are below. In each case the levels are normalized to the mean power of the signal after processing. Results are shown for  $I_1 = 1$  (i.e. clipping before interpolation),  $I_1 = 2$  and for OFDM with no clipping. Clipping after interpolation is much more effective than clipping before interpolation in reducing the dynamic range of the signal. This is particularly the case for  $CR < 6\text{dB}$ . Increasing the oversampling rate  $I_1$  further does not give significant improvement in performance.

The results in this paper are based on the structure of Fig. 1. However the second oversize IFFT could be replaced by any of the transform, upsampling and filtering arrangements commonly used in OFDM systems. So the technique can be implemented by replacing the IFFT block in an existing OFDM system with the new configuration.

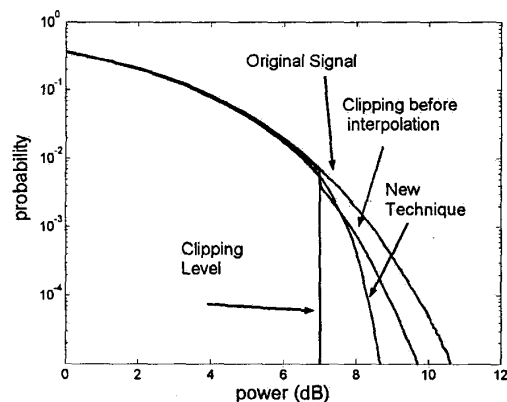


Figure 2. Cumulative distribution for clipping

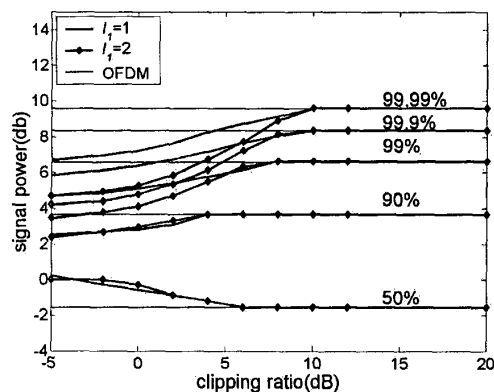


Figure 3. Distribution of instantaneous signal power for clipping before and after interpolation and undistorted OFDM.

### 3. Performance of new peak reduction technique

Figures 2 and 3 show that on the simple measure of PAPR reduction, the new technique is very effective. However the performance of the technique in a communication system also depends on the effect of the resulting in-band distortion and on how much out-of-band power will result after non-ideal amplification.

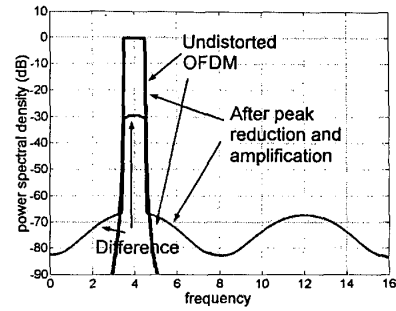
#### 3.1 Out-of-Band power

After filtering, the signal consists only of OFDM subcarriers within the nominal bandwidth of the system. Thus the spectrum of the clipped and filtered signal has precisely the same form as that of the original unclipped OFDM signal. However amplification will result in increased out-of-band power if the amplifier is not perfectly linear and/or does not have an infinite dynamic range.

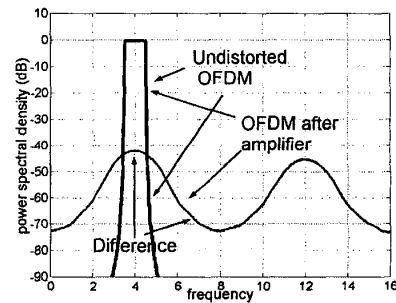
Figure 4 (a) shows the spectrum that results after modulation and amplification with an amplifier with limited dynamic range. In this case,  $CR = 6$  and  $I_1 = 2$ . The clipped and filtered baseband signal then modulates a carrier of frequency  $f_c$ , where  $f_c$  is four times the bandwidth of the OFDM baseband signal. In practice the carrier frequency would be much higher. The modulated signal is then amplified. The amplifier limits for a baseband signal 7dB above the rms baseband power but is perfectly linear up to this level. In this simulation the modulation was 4-QAM and  $N = 64$ . The spectrum calculated for an OFDM signal depends very strongly on any windowing used and also on the details of the calculation. Details of the simulation parameters are given in the appendix. The parameters were chosen to reduce out-of-band components due to symbol transitions so that the effect of intermodulation is clear.

For comparison the spectra for OFDM with no peak reduction and OFDM with clipping before interpolation are also shown. The out-of-band spectra are characterized by 'shoulders' close to the wanted band and peaks around harmonics of the carrier signal. It is easy to filter out the harmonics but more difficult to reduce the level of the shoulders. For the case shown in Fig. 4 the shoulders using the new technique are approximately 65dB down compared with 55dB for clipping before interpolation and 45dB for no peak reduction before amplification.

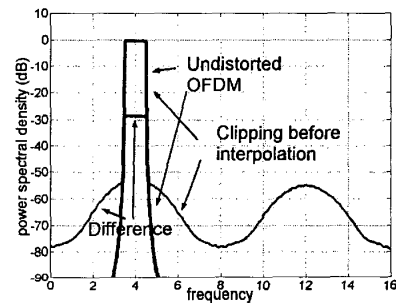
Figure 4 also show the spectra of the difference between the final signals and the original undistorted OFDM. With no clipping the in-band and out-of-band levels are closely related, with clipping they are very different. The in-band level depends on  $CR$  whereas the out-of-band level depends on the difference between  $CR$  and the amplifier limit.



(a) New technique



(b) No peak reduction



(c) Clipping before interpolation

Figure 4. Spectra when signals are amplified with an ideal limiting amplifier.

#### 3.2 In-band distortion

The new technique results in a substantial increase in the in-band component of the difference signal. However two effects make the resulting degradation less than might be expected. Clipping the signal both adds a noise like component to the signal and reduces the overall constellation size [6]. However the reduction in constellation size is corrected automatically by the receiver automatic gain control (AGC) and so does not increase the bit error rate (BER). Secondly the *effective clipping noise* that remains is added at the *transmitter* not at the receiver. Therefore in a fading channel the clipping

noise fades along with the signal, thus has much less effect than noise added at the receiver, which is not subject to fading. The precise interaction of this effect with the error-correcting coding used in all practical OFDM systems is an area of on-going research.

Figure 5 shows the BER in an additive white Gaussian noise (AWGN) channel for this case. The theoretical BER for undistorted OFDM is also shown. For 4-QAM the clipping noise cause negligible degradation in BER.

Figure 6 shows the signal to effective clipping noise ratio (SCNR) versus CR for clipping before interpolation and for clipping after interpolation with oversampling factors of 2 and 4. For  $CR > 8\text{dB}$  all of the cases converge to the same value of SCNR, which is the SCNR which would result for this amplifier with no peak limitation measures. For  $CR < 6\text{dB}$  clipping after interpolation results in a SCNR approximately 2dB higher than clipping before interpolation. The noise created by clipping is not Gaussian. The net effect of the clipping noise on overall system BER depends on the constellation size, the power of the error correcting code and whether the reduction in peak-to-mean power is used to reduce the output power rating of the amplifier or to use the existing amplifier more efficiently by increasing the transmitted power.

#### 4. Amplification with non ideal amplifiers

Practical amplifiers are never perfectly linear and the effectiveness of peak reduction techniques depends strongly on the precise characteristics of the amplifier [7]. The following simulations use the Rapp amplifier model [7] in which the output value is related to the input value by

$$y = \frac{xy_{\max}}{(1+x^{2p})^{\frac{1}{2p}}} \quad (1)$$

As  $p$  increases the amplifier becomes more linear. When this model of amplifier is used the resulting spectra are similar to the one in Fig. 4 but the level of the shoulders depends strongly on  $CR$  and  $p$ . Figure 7 illustrates this dependency.

##### 4.1 Out-of-band power

Figure 7 shows the power spectral density (PSD) at a normalized frequency offset of 70% from the carrier frequency as a function of  $p$  for  $CR = 6\text{dB}$  and  $y_{\max} = 7\text{dB}$  above the rms value of the unclipped baseband signal. The frequency offset is normalized to the signal bandwidth. Increasing  $p$  reduces the PSD in all cases, but with no peak reduction, the PSD does not fall below  $-45\text{dB}$  because a proportion of the signal samples exceed  $y_{\max}$ . All of the clipping cases substantially outperform the system with no clipping. The relative advantage increases

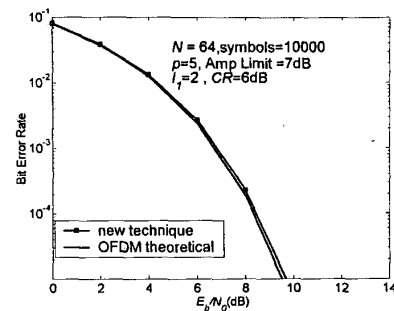


Figure 5. BER as function of  $E_b/N_0$

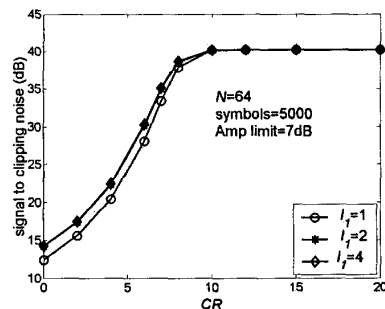


Figure 6. Clipping noise as a function of CR for ideal amplifier .

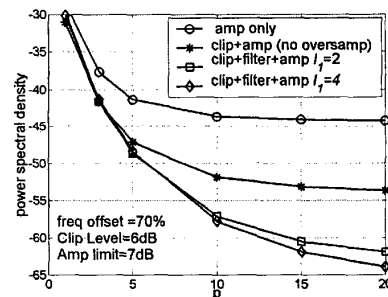


Figure 7. Power Spectral Density for non-ideal amplifier as a function of  $p$

with increasing values of  $p$ . The new clipping and filtering technique gives a lower PSD than clipping before interpolation. There is little advantage in making  $I_1 > 2$ .

##### 4.2 In-band distortion

The in-band distortion also depends on the amplifier parameter  $p$ . Figure 8 shows the effective signal to clipping noise ratio (SCNR) as a function of  $p$  for  $CR = 6\text{dB}$  and the  $y_{\max} = 7\text{dB}$ . For small values of  $p$  the distortion introduced by the amplifier dominates and there is little difference between the systems. As  $p$  increases the SCNR is highest for OFDM with no peak reduction.

Clipping after interpolation gives a slightly better SCNR than before interpolation.

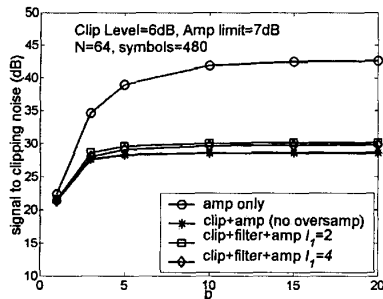


Figure 8. Signal to Clipping noise as function of  $p$

## 5. Discussion and conclusions

A new PAPR reduction technique has been described in which an interpolated version of the baseband signal is clipped and then filtered with a new form of filter. The filter consists of a forward and an inverse FFT. It is designed to remove the out-of-band noise without distorting the in-band discrete signal. It is shown that significant PAPR reduction can be achieved without any increase in out-of-band power. Some in-band distortion results but this will have negligible effect on the overall BER in most systems. The distortion has two effects: an overall shrinking of the constellation, which is automatically corrected by the receiver AGC and a noise like component. The clipping noise is added at the transmitter rather than the receiver and so fades along with the signal in a fading channel.

Simulations are presented for the combination of the system with non-ideal amplifiers. It is shown that the level of out-of-band power depends on both the degree of PAPR reduction and on the linearity of the amplifier. The new technique allows simple trade-offs between in-band distortion, amplifier back-off and amplifier linearity.

The new technique is completely compatible with other aspects of transmitter design such as windowing and filtering. It can be implemented by replacing the transmitter IFFT with an oversize IFFT, followed by the clipping and filtering circuit. The technique could also be viewed as a way of making slight changes to the input data vector in a way that reduces the PAPR. No changes are required in the receiver so can be adopted without any change to telecommunications standards.

## 6. Appendix - Simulation details

The power spectral density of an OFDM signal depends very strongly on the precise nature of transitions at symbol boundaries. Abrupt transitions result in a slow spectral

roll-off. In many OFDM systems some form of windowing or filtering is used to improve the spectrum. This paper is concerned with the out-of-band spectra due to PAPR and amplifier non-linearities so windowing was applied to smooth the transitions and increase the roll-off so that changes in out-of-band power due to the new technique could be more clearly identified. The windowing technique was the one described in [7, Chapter 2]. A cyclic prefix and postfix of length  $16T/N$  where  $T$  is the original symbol period was added to each symbol. The resulting extended symbol was windowed using a Nyquist window, with roll-off chosen so that all of the prefix and postfix were windowed but the original symbol was unchanged. Adjacent symbols were then overlapped so that the windowed cyclic prefix of one symbol was added to the windowed cyclic postfix of the preceding symbol. BER and SCNR calculations were made using only the original unwinded section of the symbol, and so were unaffected by the process.

The power spectral density was calculated using the MATLAB PSD function, which is based on the Welch periodogram method. Signal sections of 8192 samples were used and sections were overlapped by 4096 samples. A Hanning window was used.

When a signal is modulated onto a carrier the ratio of peak to rms voltage is reduced by 3dB. Amplifier back-offs are usually specified taking this into account. In this paper a similar convention was used in specifying the amplifier limit  $y_{\max}$ . For example  $y_{\max} = 7\text{dB}$  meant that the amplifier limited when the baseband signal was 7dB above the rms. value. This means that it limited for a modulated signal that is 10dB above the rms power of the modulated signal.

## 7. References

- [1] X. Li and L. J. Cimini, "Effects of Clipping and Filtering on the Performance of OFDM", *IEEE Communications Letters*, Vol. 2, no 5, pp. 131-133, May 1998.
- [2] R. O'Neill and L. N. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals," in *Proc. PMRC'95*, Sept 1995, pp71-75.
- [3] C. Tellambura, "Phase optimisation criterion for reducing peak-to-average power ratio for OFDM," *Electronics Letters*, 22 January 1998, vol 34 no 2, pp. 169-170.
- [4] J. Armstrong, Australian Provisional Patent, 2000.
- [5] D. Fraser, "Interpolation by the FFT revisited - an experimental investigation", *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol 37, no 5, May 1989.
- [6] D.Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems", *IEEE Transactions on Communications*, Vol.48, no. 10, October 2000, pp. 1755-1764.
- [7] R. Van Nee and R. Prasad, *OFDM for Multimedia Communications*, Artech House, 2000.