Unscented Kalman Filters for Polarization State Tracking and Phase Noise Mitigation

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Abstract: Joint polarization-state and phase noise tracking is demonstrated using an unscented Kalman filter. Experimentally this outperforms CMA and extended Kalman filters (EKF) and a less complex modified version outperforms CMA and EKF at higher OSNRs.
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1. Introduction

Polarization division multiplexing is a widely used technique in high-speed optical communication systems. Tracking the state of the polarization of the received signal is an important step for data recovery on both polarizations of the signal. Kalman filters have been previously demonstrated for simultaneous tracking of polarization state and phase noise [1]. As phase noise generates a non-linear observation model, an EKF has been implemented [1]. The EKF performs a linearization of the non-linear model by computing the Jacobian matrix and implements a linear Kalman filter to estimate the unknowns. However, this linearization operation causes the estimates of the EKF to be accurate only up to the 1st order under Taylor series expansion of their moments. Thus, although the Kalman filters can be considered to be optimal estimators, the EKF may give a sub-optimal convergence performance [2]. In [3], a radius directed linear Kalman filter technique was implemented for polarization-state tracking only. This technique reduces the computational complexity of the receiver system but can give errors at lower OSNRs, particularly for higher QAM modulation formats where the difference between the radii of different constellation levels is lower. Moreover, this linearized system requires different implementations for QPSK and 16-QAM modulation formats.

In this paper, we propose to use the unscented Kalman filters (UKF) [2] to improve the estimation and tracking of polarization and phase in the system. An UKF can give estimates accurate up to 3rd order under Taylor series expansion of their moments [2] and can be upgraded to higher modulation formats without drastic changes in implementations.

2. Unscented Kalman filter

The observation model for the Kalman tracking of phase and polarization state is nonlinear due to the phase term existing as an exponent in the mathematical representation of the received signal. One should not confuse this non-linear observation model with non-linear effects in the fiber. In the UKF, the distribution of state parameters (unknowns) is approximated by representing them as a set of sigma points that captures the moments under Taylor series expansion up to 3rd order. The block diagram for the UKF is shown in Fig. 1. Here, *i* is used to denote the time instance.

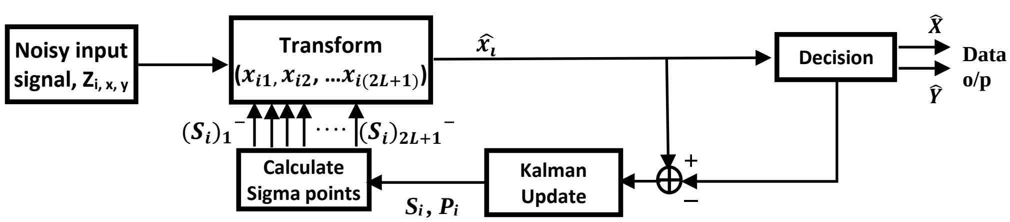

Fig. 1. Block diagram representation of Unscented Kalman filter.

The transmitted data symbols in X and Y polarizations ($\bar{x}$= [X Y]^T) can be estimated from signal $[Z_x\ Z_y]^T$ as

$$\begin{bmatrix}\hat{X}\\ \hat{Y}\end{bmatrix} = e^{j\theta}\begin{bmatrix}a+jb & c+jd\\ -c+jd & a-jb\end{bmatrix}\begin{bmatrix}Z_x\\ Z_y\end{bmatrix} \tag{1}$$

where, θ is the phase noise estimation and a, b, c, d describe the polarization state. Thus the unknown state parameter vector to be estimated is $\boldsymbol{S} = [a, b, c, d, \theta]^T$. P_i is the a posteriori estimate covariance, $(\boldsymbol{S}_i)_k^-$ is the k^{th} sigma point calculated where k = 1 to 2L+1, L is the number of state parameters to be estimated and $\hat{x}_i$ is the estimated symbol vector.

3. Modified Kalman filters

In order to avoid singularity issues in the system, the parameters a, b, c, d should be real valued. However, Kalman filters will give complex estimates for these values. To solve this issue, Reference [1] proposes to split each complex row in the algorithm matrices into two consecutive rows; the first row being the real part and second row being the imaginary part. As an alternative solution, we propose to substitute $a+jb$ with a complex value $\tilde{a}$ and $c+jd$ with $\tilde{c}$. Thus, the estimation equation becomes

$$\begin{bmatrix}\hat{X}\\ \hat{Y}\end{bmatrix} = e^{j\theta}\begin{bmatrix}\tilde{a} & \tilde{c}\\ -\tilde{c}^* & \tilde{a}^*\end{bmatrix}\begin{bmatrix}Z_x\\ Z_y\end{bmatrix} \quad (2)$$

We now find that the state parameter vector to be estimated is S = $[\tilde{a}\ \ \tilde{c}\ \ \theta]^T$ reducing the number of unknown parameters from five to three that results in the reduction of the order of matrices involved in the algorithm and thus reducing the complexity of estimation. Let us name the UKF and EKF with these reduced number of parameters as R-UKF and R-EKF respectively using Eq. (2) as the governing estimation equation. Since the Kalman filters will give complex estimations for $\tilde{a}$ and $\tilde{c}$, we avoid the need for splitting the rows and increasing the order of the matrices involved. However, to avoid singularities, some changes should be made. The Jacobian matrices become $e^{j\theta}\begin{bmatrix}Z_x & jZ_x & Z_y & jZ_y & j(a+jb)Z_x + j(c+jd)Z_y\\ Z_y & -jZ_y & -Z_x & jZ_x & j(-c+jd)Z_x + j(a-jb)Z_y\end{bmatrix}$ and $\begin{bmatrix}e^{j\theta}Z_x & e^{j\theta}Z_y & j(e^{j\theta}\tilde{a}Z_x + e^{j\theta}\tilde{c}Z_y)\\ e^{-j\theta}{Z_y}^* & -e^{-j\theta}{Z_x}^* & j(e^{-j\theta}\tilde{c}{Z_x}^* - e^{-j\theta}\tilde{a}{Z_y}^*)\end{bmatrix}$. for EKF and R-EKF respectively. For R-EKF, the data symbol estimated in Y polarization is the conjugate of the actual transmitted symbol. For both R-UKF and R-EKF, the estimates of the parameter θ will be complex. It was observed that the system avoids singularity only when that complex value is considered. Taking only the real or imaginary part, or the absolute value of the complex estimate, either leads to singularity or divergence of the filter. The explanation for this can be given that since we have combined two parameters into one, the three parameters work together to enforce the received signal transformations onto desired constellation points. This causes the Kalman filters to make the θ parameter complex valued. In other words, modified Kalman filters have fewer degrees of freedom to achieve the desired results and thus needs all the parameters to be complex. Additionally, in our system, the values for observation noise variance **R** and process noise variance **Q** giving optimum performance were found to be 10^{-1} and 10^{-8} for EKF and UKF, and 5×10^{-1} and 10^{-4} for R-EKF and R-UKF respectively. This increased variance leads to a lower performance at lower OSNRs.

4. Experiment and results

A 5-GBaud QPSK electrical signal was generated using arbitrary waveform generator (Tektronix AWG7102). The signal was oversampled to 10 GSa/s and then modulated using an IQ modulator (Sumitomo T.SBXH1.5-20PD-ADC). Polarization multiplexing was emulated using a delay line, polarization beam splitters and combiners. In the case of back-to-back configuration, the amplified signal was directly fed to an optical hybrid (Kylia MINT-0060) and detected using four balanced photodiodes (Finisar BPDV2020R). The outputs of the coherent receiver were connected to a 40-GSa/s, 28-GHz bandwidth DSO (Keysight DSO-X-92804A). The algorithms were run on the signal as offline DSP after resampling the signal to 10 GSa/s. To investigate behavior in transmission, the signal was also sent over 10×80km SSMF spans at various launch powers. Carrier frequency offset in the system is compensated using spectrum based method in all the cases before passing the samples to the test algorithms. Table 1 shows the number of complex multiplications required for each algorithm to detect one symbol in each polarization.

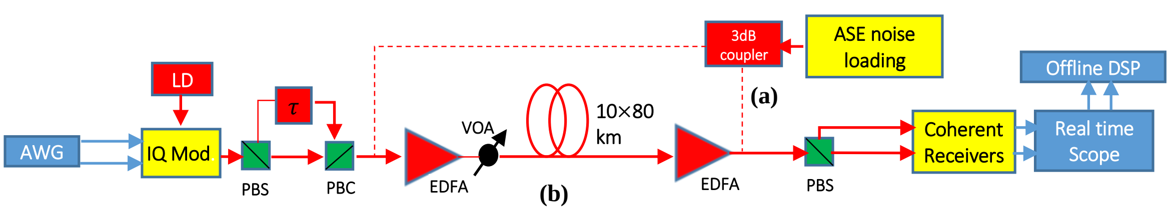

Fig. 2. Experimental setup in a) back-to-back configuration b) 10×80 km transmission link configuration.

Algorithm	Name	Number of complex multiplications per symbol
UKF	Unscented KF	140
R-UKF	Reduced-Unscented KF	90
EKF	Extended KF	95
R-EKF	Reduced-Extended KF	78

Table 1. Number of complex multiplications required by algorithms to detect one symbol on each polarization.

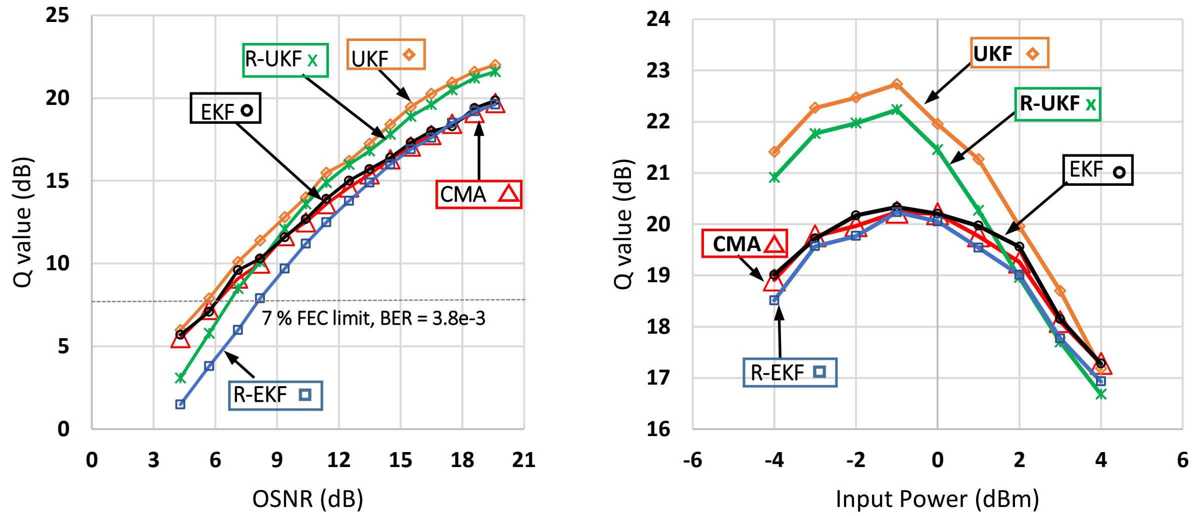

Fig. 2. a) Q-value vs. OSNR in back-to-back configuration b) Q-value vs. injected power level over 800 km.

Fig. 2(a) shows that the UKF and R-UKF both outperform the EKF, showing an increase in Q-value of 2.5-dB and 2-dB respectively at 20-dB OSNR. At low OSNR, the UKF converges to the performance of EKF, while the R-UKF shows a 0.6-dB OSNR penalty at the 7% FEC threshold. Where sufficient errors were measured (Q < 8.5-dB), the Q-value calculated from the constellation variance is equal to the Q-value calculated from the finite BER. Fig. 2(a) also shows the UKF and R-UKF performance compared with conventionally used constant modulus algorithm (CMA) and Viterbi and Viterbi phase estimation (VVPE) algorithm [4]. In this case also, the UKF and R-UKF give up to 2.6-dB and up to 2.1-dB improvement at 20-dB OSNR respectively. This improvement is observed because of a more intelligent update of the Kalman gain using the *a priori* and *a posteriori* estimate error covariance against the constant step size of CMA. This not only gives a performance improvement but also gives faster convergence to UKF and R-UKF than CMA. These *a priori* and *a posteriori* estimate error covariances are captured more accurately by the unscented transformation [2], and hence we get the benefit of UKF and R-UKF over EKF. The improvement over CMA (41 taps) is not observed in EKF since the linearization process disturbs the accuracy of the estimate covariances.

Fig. 2(b) shows that each algorithm has the same optimal launch power, and converge to similar performance. The performance tracking is lost due to non-linear effects at high launch powers. This is to be expected, as none of the algorithms are capable of compensating for fast state changes caused by the non-linear distortions in the fiber. At peak Q, we expect the signal to have a high OSNR, leading to the increased performance of the UKF implementations over other algorithms at optimal launch power. Overall, these results indicate that the EKF may not be a useful alternative to CMA+VVPE, while the performance improvement from using the UKF may come at the cost of increased complexity. However, a direct comparison of the complexity of CMA versus Kalman filter equalization is not straightforward due to the decision-direction nature of Kalman filter implementation.

5. Conclusions

UKF outperforms EKF at the cost of complexity, the UKF outperforms CMA+VVPE especially at moderate and high OSNRs which translates to better peak Q. The R-UKF outperforms CMA+VVPE and EKF at moderate and higher OSNRs, but gives penalty at lower OSNRs. The R-EKF is less-complex than the EKF, but performs similarly to EKF only at high OSNRs giving a penalty otherwise.

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6. References

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