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On Approximation Of Outage Probability In CDMA Cellular Networks

Syed Bilgrami and Jaunty Ho

MONASH UNIVERSITY

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Syed Bilgrami and Jaunty Ho
Department of Electrical and Computer Systems Engineering ,
Monash University, Clayton VIC 3800, Australia.
Email: {syed.bilgrami, jaunty.ho}@eng.monash.edu.au

***Abstract*— In this paper, we investigate an approximation for exact outage probability for code division multiple access (CDMA) Cellular Mobile Networks. We compare Poisson-Gaussian Method with large deviation approximation methods like *Crammer's, Chernoff and Bahadur-Rao* against simulated outage probability. Our results indicates that the Gaussian approximation underestimates the outage probability. Chernoff bound captures the correct exponent in the probability of outage decay. At higher arrival intensities, the three approximation methods seems to give approximately equal performance. Outage probabilities may increase as the interference channel load may increase.**

***Index Terms*— Code Division Multiple Access (CDMA), Outage Probability, Quality of Service, Poisson Traffic.**

I. INTRODUCTION

Recently, one the most popular form of spread spectrum multiple access technique is regarded as Code Division Multiple Access (CDMA). In CDMA, cellular users may share the common spectrum simultaneously [2]. In FDMA and TDMA cellular networks, an outage may be regarded as a result of high propagation loss occurring during a handoff situation [9]. The outage in CDMA, is far more detrimental because the interference experienced at base station in CDMA, may be higher and this may cause overloading due to too may adjacent cellular active users, transmissions in adjacent cellular networks. Spatial models are described in [2] which investigate number of available channels in TDMA or FDMA cellular networks and interference experienced at base station in CDMA networks.

Quality of Service (QoS) comprises of factors such as bit errors, frame errors or packet errors, network delay or bit-rate. We consider the case where require a certain level of bit energy to noise ratio to be achieved or exceeded [9]. If this level of bit energy to noise drops below a minimum threshold we may say an outage event have occurred.

Two important factors in network dimensioning are numerical complexity of algorithm and statistical characterization of interference function. Realistic network dimensioning may involve approximation of large number of traffic scenarios. Some approximation methods which require a relatively low number of computation giving accurate results, where as other methods require only mean and variance of interference function. The contribution of this work is an extension, to the previous [9], [2]. We consider approximation of outage probabilities that are very small i.e. 10^{-4} and below referred to as *tail probabilities* in probability theory. In spread spectrum system [9] all users share the same frequency, hence the interference seen at base station is summation of interference contributed by individual users in the cellular system. However in order to generalize the model, we applied these results to higher values of outage probabilities, channel loads and arrival intensities. Our results indicates that the outage probability may increase as the channel load is increased. At higher values of arrival intensities λ_1 and small outage probabilities Chernoff, Bahadur-Rao and Gaussian Approximation Methods indicates the same equal performance.

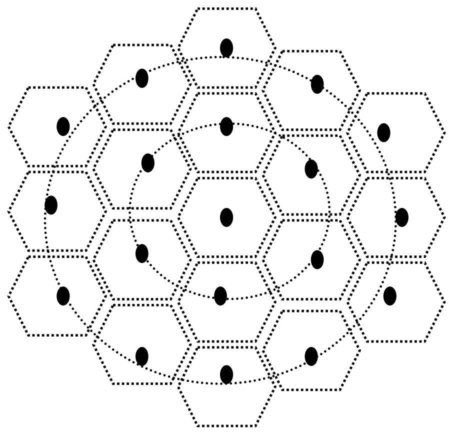

Fig. 1. Hexagon of 19 Cells.

II. APPROXIMATION OF OUTAGE PROBABILITY

Outage probability may be defined as when the interference level at the reference base station exceeds a threshold that makes it impossible for the user's to communicate i.e. if it is greater or equal to the minimum required established connectivity level. A single user 1 can establish connectivity if and only if [9]

$$\frac{E_b}{N_0} \leq \frac{\frac{\beta_1 K}{T}}{IL + \theta K} \quad (1)$$

Where K is the available spectrum, IL represents the total interference seen at reference base station, $\frac{E_b}{N_0}$ represents the minimum required bit energy to noise ratio, T is the determined data rate and β_1 is the received power of user 1 and θ is the background noise coefficient. In [2] it is presented that class 1 user can establish connectivity if and only if outage probability is:

$$P(IL \geq CO) \tag{2}$$

where IL is the total interference level seen at the reference base station. If we consider the spatial Poisson point pattern as mentioned [2] , which is based on assumptions like the model as being a snapshot in time of a real system and does not include constraints related with time dynamics. Furthermore, they have considered a cluster made up of 19 base stations (as shown in figure 1) spatially arranged in hexagonal-shaped cells with adjacent base stations separated by a distance 'd'. All users are are assumed to be in soft handover with reference base station position in the center of the 19 cell cluster [2]. In realistic approach all users are in soft handover with reference base station and we would like to consider this effect in our analysis. The propagation environment of users are outdoor environment and we assume effects such as path loss and shadowing effects. We ignore effects related with multi-path effects. As per Central Limit Theorem we consider that the system is large enough to ignore correlations between individual signature sequences. With these assumptions and [2], [5] we may say that the interference contributed to the reference base station from this class 1 user is given by

$$min\left[\beta_1, \beta_1 \frac{d_0^{-\gamma} L_0}{d_1^{-\gamma} L_1}, \beta_1 \frac{d_0^{-\gamma} L_0}{d_2^{-\gamma} L_2}\right] \tag{3}$$

Where β_1 is the received power of class 1 user and γ represents the path loss exponent for environment propagation effects. d_0, d_1, d_2 represents the distance from user to base station 0, base station 1 and base station 2 respectively. L_0, L_1, L_2 are independent and identically distributed random variables representing the shadowing effects of the user signal to the reference base station and that of the user to closest base station. We may write [5] $L_i = 10_i^G$ where $G_i \approx N(0, \sigma^2)$ and $i = 0, 1, 2$.

Suppose $i_1(X, H)$ be a deterministic function that represents the interference contributed by a $class_1$ user located at position X to the reference base station. Where H is a stochastic process defined with values representing the effects of shadowing. Consider in each of the divisions, each class of user forms a spatial Poisson point pattern [2], [5] with the same intensity. Then the total interference (IL) at the reference base station contributed by class 1 user is given by

$$IL = \sum_{X \epsilon \pi_1(\lambda_1)} i_1(X, H) \tag{4}$$

A. Lemma 1

The moments of the total interference seen at the reference base station from the class 1 users can be calculated by

$$E((IL)^k) = \lambda_1 \beta_1^k d^2 c_1(k) \tag{5}$$

Where $k = 1, 2, 3, ...$ and β_1 represents the received power of class 1 and d is the distance separating the two adjacent base stations. The detailed derivation was presented as [**?**], [5]. We may present an extension to their analysis, that the outage probability of class 1 user can be written as

$$P\left(\sum_{j=1}^{J} \sum_{X \epsilon \pi_j(\lambda_j)} i_j(X, H) \geq CO_1\right). \tag{6}$$

In above equation summation is commonly referred to as the interference function and CO_1 represents the interference threshold for class 1 user. The spatial Poisson point [2] pattern π_j depends upon the arrival intensity λ_j.

III. Gaussian Method

This is the most common quick and simple method used in literature to approximate the interference contributed by user [1], [3]. This approximation requires only the mean and the variance of the interference function and its usage can be justified by well known Central Limit Theorem [4]. Suppose Lindeberg conditions [8] are satisfied, then Central Limit Theorem can be applied to equation (5) and the Gaussian approximation for outage probability of class 1 can written as

$$P\left(\sum_{j=1}^{J} \sum_{X \epsilon \pi_j(\lambda_j)} i_j(X, H) \geq CO_1\right) \approx Q\left(\frac{CO_1 - k^1}{\sqrt{k^2}}\right). \tag{7}$$

The Q-function for the above standard Gaussian distribution with zero mean and unit variance can be denoted as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du.$$

IV. Large Deviation Methods

In this section, we will approximate the outage probability equation (5) by applying methods [6], [7]. In the following methods we will refer the total interference seen at reference bases station as I.

A. Lemma 1

Suppose a set of independent and identically distributed random variables $X_1, X_2, ..., X_N$. If $E(e^{sX_1}) < \infty$ for all $s\epsilon R$ then Crammer's Theorem states that

$$\lim_{N\to\infty} \frac{1}{N}\, log_e\, P\left(\frac{1}{N}\sum_{i=1}^{N} X_i > C\right) = I(C). \tag{8}$$

For any $N > 0$, [6] states that

$$\frac{1}{N}\, log_e\, P\left(\frac{1}{N}\sum_{i=1}^{N} X_i > C\right) \leq I(C) \tag{9}$$

Where $I(C) \approx inf_{s\epsilon R}[-sC + log_e E(e^{sX_1})]$. In spatial Poisson model [2], J Poisson Point patterns $\pi_1, \pi_2, ..., \pi_j$ can be represented as superposition of N independent and identically distributed Poisson point patterns as $N\lambda_1, N\lambda_2, ..., N\lambda_j$ with same arrival intensity λ_j. For the case of equation (5) and any $N > 0$ we may write equation (8) as

$$\frac{1}{N}\, log_e\, P\left(\frac{1}{N}\sum_{j=1}^{J} \sum_{X \epsilon \pi_j(N\lambda_j)} i_j(X, H) > CO_1\right) \leq I_1(CO_1) \tag{10}$$

Where

$$I_1(CO_1) \approx inf_{s_1 \epsilon R}[-s_1 CO_1 + \sum_{j=1}^{J} log_e M_j(s_1)]. \quad (11)$$

We may write the moment generating function for the interference function as $M_j(s_1)$ where

$$M_j(s_1) = E\Bigg(exp\Bigg(s_1 \sum_{X \epsilon \pi_j(\lambda_j)} i_j(X, H)\Bigg)\Bigg). \quad (12)$$

Using the computation developed in [2], equation (9) can be written as

$$P\Bigg(\frac{1}{N}\sum_{j=1}^{J}\sum_{X \epsilon \pi_j(N\lambda_j)} i_j(X, H) \geq CO_1\Bigg) \leq e^{NI_1(CO_1)}. \quad (13)$$

If we assume a uniform bound over the total interference [2] then $I_1(CO_1)$ in equation (12) can be written similarly, as equation (10).

B. Lemma 2

Let $X_1, X_2, ..., X_N$ be independent and identically distributed random variables. If we assume that $M(s) = E(e^{sX_1}) < \infty$ for all $s \epsilon R$ then

$$\lim_{N\to\infty} \frac{1}{N}\, log_e\Bigg((\theta s_a \sqrt{2\pi N}) P\Bigg(\frac{1}{N}\sum_{i=1}^{N} X_i \geq C\Bigg)\Bigg) = I(C) \quad (14)$$

Where

$$I(C) = inf_{s \epsilon R}[-sC + log_e M(s)], \quad (15)$$

$$\theta^2 = \frac{d^2(M(s))}{ds^2},$$

Where $s = s_a$ and

$$s_a = arginf_{s\epsilon R}[-sC + log_e M(s)].$$

Applying above equations [7] to equation (5) we may write

$$P\Bigg(\frac{1}{N}\sum_{j=1}^{J}\sum_{X \epsilon \pi_j(N\lambda_j)} i_j(X, H) \geq CO_1\Bigg) \approx \frac{e^{NI_1(CO_1)}}{s_1\theta_1\sqrt{2\pi N}}. \quad (16)$$

In equation (15), $I_1(CO_1)$ can be similarly computed as presented equation (10). Computation of θ in above equation may increase in number of computation as compared to equation (12). s_1 in equation (15), can be obtained by minimizing s_1 in equation (10).

V. Simulation

In this section, we compute the outage probability of two class cellular system, using the three approximation methods described in previous sections (Figure 3). We simulate outage probabilities as presented in Figure 2 and Figure 3. For Figure 2 and Figure 3 simulation results are obtained using Monte-Carlo simulations. We include 95% confidence interval for each simulated outage probability. We conducted $5x10^5$ independent simulation for each outage probabilities and then we compute the results over the large range of arrival intensities λ_1 from 1.5 to 13.5. We consider scaling factor $N \approx 1$ for Chernoff and Bahadur-Rao approximations. We consider CO_1, λ_2, β_1 and β_2 as fixed values. Figure 2 presents a comparison of simulated outage probabilities with equation (6) presented. Figure 3 shows that the performance of equation (6) is same as that of Bhadur-Rao. In figure 2 we present the comparison of outage probability with Normalized Load per channel(λ_1/μ) where μ is the service rate in $M/M/\infty$ network.

Figure 2 presents an important result that outage probability may increase as Normalized Load per interference Channel is increased. The relationship between network load and outage probability may contribute to plan and design networks such as to increase network throughput, service availability and minimize cellular outage event probabilities.

Figure 3 provides an important result between arrival intensities and outage event probabilities. In reality, large cellular networks with large number of users may have higher values of outage probability in the range 0.9 to 0.01 [10]. As the arrival intensities for class 1 are increased approximately 6 or above, the three performance models presented are approximately equal in outage performance. Figure 3 indicates that at higher class intensities (if we consider most outage events) the three models presented, are approximately equal. This may help to plan a network as to minimize outage probability, we need to consider smaller and smaller class intensities. Gaussian Approximation does not estimates *tail probabilities* very well for smaller class intensities. We compare the results with simulated outage probabilities and presents that equation (6) and Bahadur-Rao methods are relatively simple and accurate to compute outage probabilities until the arrival intensities of particular class are below or equal to 6. Chernoff presents an upper bound on the outage probability.

VI. Conclusion And Future Work

The comparison presented, can be applied to network planning and dimensioning. We may compute network load and throughput to approximate relative outage probabilities to characterize the system parameters providing total service availability and guaranteed quality of service to users. Figure 2 presents an insight relationship between service outage and input traffic load per interference channel. Figure 3 presents a comparison between the three methods and their relationship with higher and lower class intensities. For lower intensities and tail probabilities Bahadur-Rao and equation (6) are accurate methods. But for higher class intensities and higher outage events (realistic case) the three methods are approximately equal, accurate and provide insight into dimensioning of guaranteed quality of service in CDMA cellular network.

The estimation of system performance parameters may help network planners to provide guaranteed quality of service. In future, we would like to like to extend and generalize this model and our analysis towards approximation of interference channel capacity. We will approximate network throughput and resource utilization for large spatial spread spectrum cellular networks.

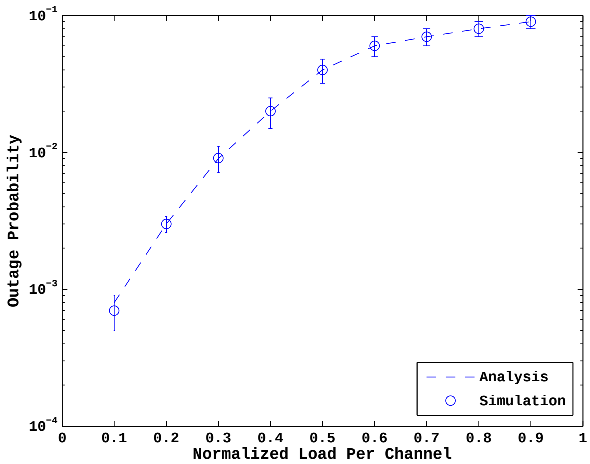

Fig. 2. Outage Probability of Class 1 user V/s Load per interference Channel.

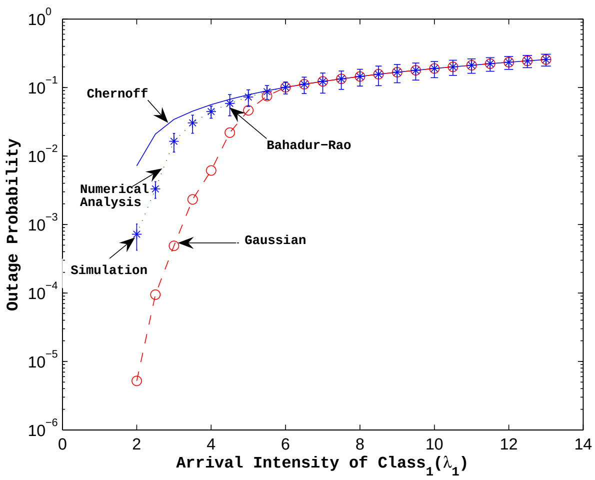

Fig. 3. Outage Probability of Class 1 user V/s Approximation Methods.

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